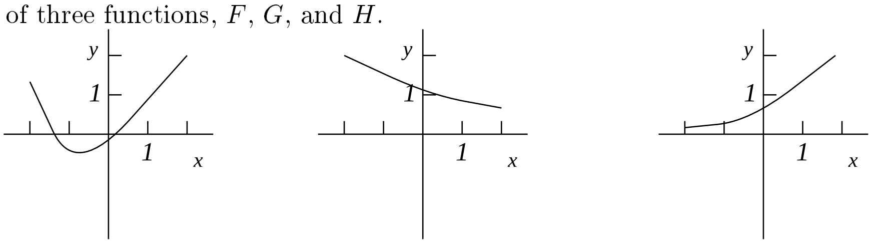
Suppose V is the curve in the plane consisting of points whose coordinates satisfy $y = x^2$ and W is the curve in the plane consisting of points whose coordinates satisfy $x = y^4$ and U is the curve in the plane consisting of points whose coordinates satisfy $y = x^3 - 2x$.
a) Describe how to find the area enclosed by V and W. Describe how to find the area enclosed by V and U.
b) Suppose the area enclosed by V and W is called A. Describe how to compute the volume swept out if A is revolved about the $x-$axis. And now the $y-$axis.
c) Suppose a solid object is constructed whose base is A and whose cross-sections perpendicular to the x axis are squares with sides on the (x, y)-plane. Describe how to compute the object's volume.

Finding **convenient** and **verifiable** overestimates of the magnitude of functions has occurred several times in this course (For example, in using the Trapezoid Rule, Simpson's Rule, and Taylor's Theorem).
Here are graphs of three functions, F, G, and H.



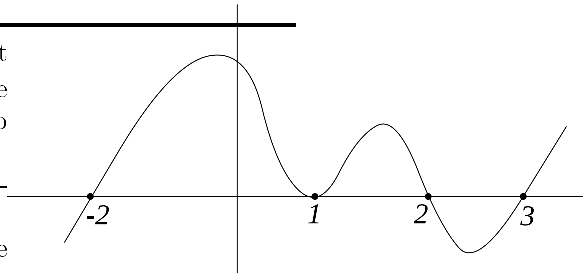
F *G* *H*

Find overestimates of the magnitude (the absolute value) of the following functions on the following intervals. Be sure to have some justification of each assertion. Note: 10^{10} is **not** a good answer to any of these questions!
$F(x)$ on the interval $[0, 2]$; $G(x)$ on the interval $[-1, 1]$; $x^3 H(x)$ on the interval $[0, 2]$; $3\cos(x^7) + 5G(x)\sin(e^x)$ on the interval $[0, 2]$; $8G(x)(5 + x)^{-1/7}$ on the interval $[-1, 2]$; $5H(x)(5 + x)^{-1/7}$ on the interval $[-1, 2]$.

How would you find antiderivatives of the following functions? (I may not be able to do some of these!)

$$\sin(e^x)e^{2x} \quad \sin\left(e^{2x}\right)e^x \quad \sin(e^x)e^x \quad \sin(e^x)x \quad \sin(x)e^x$$

Here is a graph of a function, Q. **All** that you know about Q is what is on the graph*. Now suppose I want to analyze solutions of the differential equation $y' = Q(y)$ subject to the initial condition $y(0) = y_0$.



• Please draw (approximately) the direction field association with $y' = Q(y)$ for $-2 \le x \le 2$ and $-2 \le y \le 3$.
• There are a few values of y_0 for which solution curves are horizontal lines. What are those values of y_0?
• Suppose $y(x)$ is a solution curve to $y' = Q(y)$ with $y(0) = -1$. Sketch this solution curve. Then what are $\lim_{x\to\infty} y(x)$ and $\lim_{x\to-\infty} y(x)$?
• Suppose $y(x)$ is a solution curve to $y' = Q(y)$ with $y(0) = 2.5$. Sketch this solution curve. Then what are $\lim_{x\to\infty} y(x)$ and $\lim_{x\to-\infty} y(x)$?

How would you find antiderivatives of the following functions? (At least one of these would take a very long time to finish!)

$$\frac{x^2}{(x+1)(x+2)} \qquad \frac{(x+1)(x+2)}{x^2} \qquad \frac{x^2}{(x^2+1)(x^2+2)} \qquad \frac{1}{x^{1/3}(2+x^{1/3})} \qquad \frac{1}{x^{1/4}(2+x^{1/3})}$$

If $xy' = e^y$ and if $y(1) = 0$, what's the largest possible domain of y when expressed as explicitly as possible as a function of x?

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* If this makes you really nervous, then just imagine $Q(w)$ is $\frac{1}{22}(w+2)(w-1)^2(w-2)(w-3)$ which should calm you down a bit.

Order these from easiest to do to hardest to do:

$$\int (\sin x)^2\,dx \quad \int (\sin x)^3\,dx \quad \int (\sec x)^2\,dx \quad \int (\sec x)^3\,dx \quad \int (\tan x)^2\,dx \quad \int (\tan x)^3\,dx$$

Compute $\int_{-1}^{1} \sin(e^x)\,dx$ to an accuracy of $\pm .001$.

Does the sequence $\left\{\left(1+\frac{1}{n}\right)^n\right\}$ converge? Does the series $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^n$ converge?

Suppose $F(0)=2$ and $F'(0)=3$ and $F''(0)=-9$ and $F'''(0)=3$ and $F''''(0)=2$. What is $T_3(x)$, the third degree Taylor polynomial of F centered at 0? Given this information, what can you say about $R_3(100)$? What would you expect to be true about $R_3(x)$ when x is very small?

Compute $\int_0^{1/2} \frac{x^3}{3+x^6}\,dx$ to an accuracy of $\pm .001$.

The hyperbolic sine of x, $\sinh x$, is defined by the equation $\sinh x = \frac{e^x - e^{-x}}{2}$. Use the power series for the exponential function to get a power series for $\sinh x$.

Create a function J defined for $0 < x < 2$ so that $J(x) > 0$ for all those x's and $\lim_{x\to 0^+} J(x) = +\infty$ and $\lim_{x\to 2^-} J(x) = +\infty$ and $\int_0^1 J(x)\,dx$ converges and $\int_1^2 J(x)\,dx$ diverges.

Describe how to compute $\sin(.2)$ to an accuracy of $\pm 10^{-20}$. That is, write down an explicit sum which is equal to $\sin(.2)$ within this accuracy, and describe why you know that your statement is true.

What is the radius of convergence of $\sum_{n=0}^{\infty} \frac{(3n)!}{n!(2n)!}x^n$?
