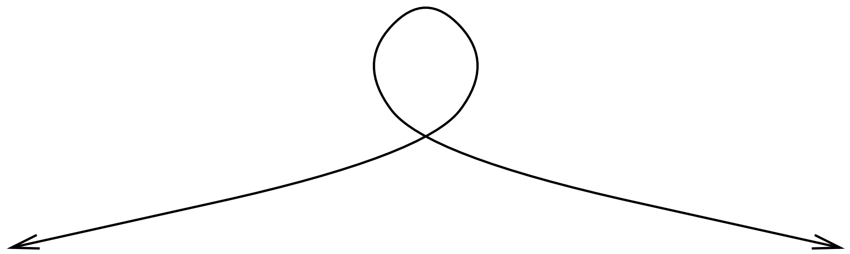
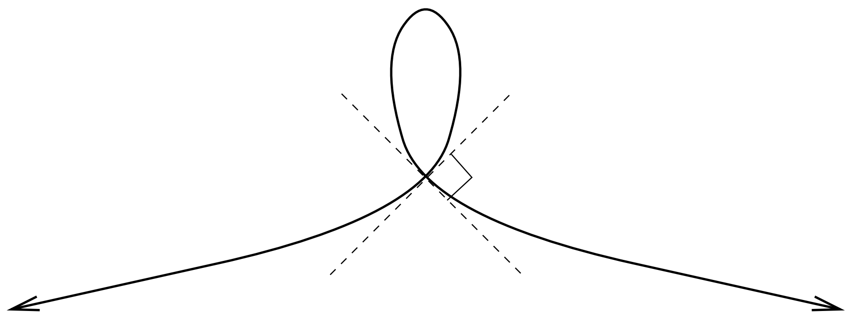
Problems, Week 10

1. If A is a positive number, the parametric curve $\begin{cases} x = t^3 - t \\ y = \dfrac{A}{1+t^2} \end{cases}$ looks like:

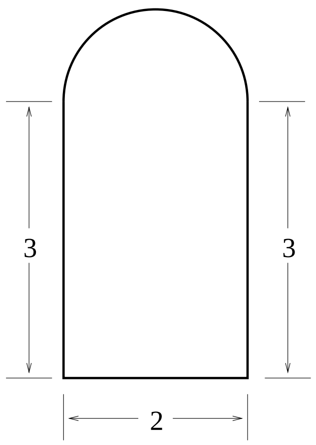


There's one value of the number A so that the "self-intersection" of the curve is perpendicular and the picture looks like:



Find that value of A.

2. A window is in the shape of a $2'$ by $3'$ rectangle surmounted by a semicircle*. Describe the boundary of the window as $r = f(\theta)$ in a polar coordinate system whose location and orientation you specify. Note that $f(\theta)$ will need to be defined "piecewise".



* It is traditional for a calculus course to have a problem discussing this sort of window, including the rather uncommon word "surmount" as part of the description. We honor tradition here!

3. A point P travels on the parabola $y = x^2$.

a) Give parametric formulas for the location of P where the parameter is the first coordinate of the point P.

b) Give parametric formulas for the location of P where the parameter is the second coordinate of the point P. The parametric formulas will have to be given "piecewise".

c) Give parametric formulas for the location of P where the parameter is the angle that the ray from the origin to P makes with the positive x-axis.

d) Give parametric formulas for the location of P where the parameter is the angle that the ray from the point $(0,1)$ to P makes with the ray from $(0,1)$ to the origin.

e) Give parametric formulas for the location of P where the parameter is the distance from the origin to P. The parametric formulas again will have to be given "piecewise".

4. Find the Cartesian coordinates of <u>all</u> points of intersection of the curves with polar equations $r^2 = 4\sin\theta$ and $r = 1 - \sin\theta$. (Warning: $(0,\theta_1)$ and $(0,\theta_2)$ represent the same point *always*; also, so do (r,θ) and $(-r,\theta \pm \pi)$.)