
Math 152, Fall 2011, Formula Sheet for Exam 2

$$\begin{aligned} \sin(0) = 0 ; \quad \sin(\pi/6) = 1/2 ; \quad \sin(\pi/4) = \sqrt{2}/2 ; \quad \sin(\pi/3) = \sqrt{3}/2 ; \quad \sin(\pi/2) = 1 \\ \cos(0) = 1 ; \quad \cos(\pi/6) = \sqrt{3}/2 ; \quad \cos(\pi/4) = \sqrt{2}/2 ; \quad \cos(\pi/3) = 1/2 ; \quad \cos(\pi/2) = 0 \end{aligned}$$

$$\cos^2 x + \sin^2 x = 1 ; \quad 1 + \tan^2 x = \sec^2 x ; \quad 1 + \cot^2 x = \csc^2 x$$

$$\sin(2x) = 2 \sin x \cos x ; \quad \cos(2x) = \cos^2 x - \sin^2 x$$

$$\cos^2 x = \frac{1}{2}(1 + \cos(2x)) ; \quad \sin^2 x = \frac{1}{2}(1 - \cos(2x))$$

$$\sin A \cos B = \frac{1}{2}[\sin(A - B) + \sin(A + B)]$$

$$\sin A \sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$$

$$\cos A \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$$

$$\int \sec x \, dx = \ln |\sec x + \tan x| + C ; \quad \int \csc x \, dx = -\ln |\csc x + \cot x| + C$$

If T_N , M_N , S_N are the Trapezoidal, Midpoint and Simpson's approximations, then

$$T_N = \frac{\Delta x}{2}[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{N-2}) + 2f(x_{N-1}) + f(x_N)] ;$$

$$M_N = \Delta x[f(c_1) + f(c_2) + \cdots + f(c_N)] \text{ where } c_j = (x_{j-1} + x_j)/2 ;$$

$$S_N = \frac{\Delta x}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{N-2}) + 4f(x_{N-1}) + f(x_N)] .$$

If $I = \int_a^b f(x) \, dx$ then

$$|T_N - I| \leq \frac{K_2(b-a)^3}{12N^2} , \quad |M_N - I| \leq \frac{K_2(b-a)^3}{24N^2} , \quad |S_N - I| \leq \frac{K_4(b-a)^5}{180N^4} .$$

The length of the curve $y = f(x)$, $a \leq x \leq b$ is equal to $\int_a^b \sqrt{1 + (f'(x))^2} \, dx$.

The area of the surface obtained by rotating the curve $y = f(x)$, $a \leq x \leq b$ about the x -axis is equal to $\int_a^b 2\pi f(x) \sqrt{1 + (f'(x))^2} \, dx$.

The length of the parametric curve $(x(t), y(t))$, $a \leq t \leq b$ equals $\int_a^b \sqrt{(x'(t))^2 + (y'(t))^2} \, dt$.

If a curve is given in polar form by $r = f(\theta)$ then the area bounded by $r = f(\theta)$, $\theta = \alpha$ and $\theta = \beta$ is $\int_\alpha^\beta \frac{1}{2} r^2 \, d\theta = \int_\alpha^\beta \frac{1}{2} (f(\theta))^2 \, d\theta$. The length of the polar curve $r = f(\theta)$ between $\theta = \alpha$ and $\theta = \beta$ is $\int_\alpha^\beta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \, d\theta = \int_\alpha^\beta \sqrt{(f(\theta))^2 + (f'(\theta))^2} \, d\theta$.

Newton's Law of Cooling is given by $\frac{dT}{dt} = -k(T - T_0)$, where T is the temperature and T_0 is the ambient temperature. The balance P in an annuity is given by $\frac{dP}{dt} = r(P - N/r)$, where r is the interest rate and N is the rate of withdrawal.

The n th Taylor polynomial of $f(x)$ with center a is $T_n(x) = \sum_{k=0}^n \frac{f^{(k)}(a)}{k!} (x - a)^k$.

If $|f^{(n+1)}(u)| \leq K$ for all u between a and x , then $|f(x) - T_n(x)| \leq K \frac{|x - a|^{n+1}}{(n+1)!}$.