Math 152, Spring 2009, Formula Sheet for Exam 2

$$\sin(0) = 0\ ;\quad \sin(\pi/6) = 1/2\ ;\quad \sin(\pi/4) = \sqrt{2}/2\ ;\quad \sin(\pi/3) = \sqrt{3}/2\ ;\quad \sin(\pi/2) = 1$$
$$\cos(0) = 1\ ;\quad \cos(\pi/6) = \sqrt{3}/2\ ;\quad \cos(\pi/4) = \sqrt{2}/2\ ;\quad \cos(\pi/3) = 1/2\ ;\quad \cos(\pi/2) = 0$$

$$\cos^2 x + \sin^2 x = 1\ ;\quad 1 + \tan^2 x = \sec^2 x\ ;\quad 1 + \cot^2 x = \csc^2 x$$
$$\sin(2x) = 2\sin x\cos x\ ;\quad \cos(2x) = \cos^2 x - \sin^2 x$$
$$\cos^2 x = \frac{1}{2}\big(1 + \cos(2x)\big)\ ;\quad \sin^2 x = \frac{1}{2}\big(1 - \cos(2x)\big)$$
$$\sin A\cos B = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$$
$$\sin A\sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$$
$$\cos A\cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C\ ;\quad \int \csc x\,dx = \ln|\csc x - \cot x| + C$$

If T_N, M_N, S_N are the Trapezoidal, Midpoint and Simpson's approximations, then

$$T_N = \frac{\Delta x}{2}[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{N-2}) + 2f(x_{N-1}) + f(x_N)]\ ;$$
$$M_N = \Delta x[f(c_1) + f(c_2) + \cdots + f(c_N)] \text{ where } c_j = (x_{j-1} + x_j)/2\ ;$$
$$S_N = \frac{\Delta x}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{N-2}) + 4f(x_{N-1}) + f(x_N)]\ .$$

If $I = \int_a^b f(x)\,dx$ then:

$$|T_N - I| \le \frac{K_2(b-a)^3}{12N^2}\ ;\quad |M_N - I| \le \frac{K_2(b-a)^3}{24N^2}\ ;\quad |S_N - I| \le \frac{K_4(b-a)^5}{180N^4}\ .$$

$$\text{length} = \int_a^b \sqrt{1 + [f'(x)]^2}\,dx\ ;\quad \text{surface area} = 2\pi\int_a^b f(x)\sqrt{1 + [f'(x)]^2}\,dx\ .$$

The nth Taylor polynomial of $f(x)$ with center c is $T_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(c)}{k!}(x-c)^k$.

If $|f^{(n+1)}(u)| \le K$ for all u between c and x, then $|f(x) - T_n(x)| \le K\frac{|x-c|^{n+1}}{(n+1)!}$.