Math 152, Spring 2010, Formula Sheet for Exam 2

$$\sin(0)=0\ ;\quad \sin(\pi/6)=1/2\ ;\quad \sin(\pi/4)=\sqrt{2}/2\ ;\quad \sin(\pi/3)=\sqrt{3}/2\ ;\quad \sin(\pi/2)=1$$
$$\cos(0)=1\ ;\quad \cos(\pi/6)=\sqrt{3}/2\ ;\quad \cos(\pi/4)=\sqrt{2}/2\ ;\quad \cos(\pi/3)=1/2\ ;\quad \cos(\pi/2)=0$$
$$\cos^2 x+\sin^2 x=1\ ;\quad 1+\tan^2 x=\sec^2 x\ ;\quad 1+\cot^2 x=\csc^2 x$$
$$\sin(2x)=2\sin x\cos x\ ;\quad \cos(2x)=\cos^2 x-\sin^2 x$$
$$\cos^2 x=\frac{1}{2}\big(1+\cos(2x)\big)\ ;\quad \sin^2 x=\frac{1}{2}\big(1-\cos(2x)\big)$$
$$\sin A\cos B=\frac{1}{2}[\sin(A-B)+\sin(A+B)]$$
$$\sin A\sin B=\frac{1}{2}[\cos(A-B)-\cos(A+B)]$$
$$\cos A\cos B=\frac{1}{2}[\cos(A-B)+\cos(A+B)]$$
$$\int\sec x\,dx=\ln|\sec x+\tan x|+C\ ;\quad \int\csc x\,dx=\ln|\csc x-\cot x|+C$$

If T_N, M_N, S_N are the Trapezoidal, Midpoint and Simpson's approximations, then

$$T_N=\frac{\Delta x}{2}[f(x_0)+2f(x_1)+2f(x_2)+\cdots+2f(x_{N-2})+2f(x_{N-1})+f(x_N)]\ ;$$
$$M_N=\Delta x[f(c_1)+f(c_2)+\cdots+f(c_N)]\text{ where }c_j=(x_{j-1}+x_j)/2\ ;$$
$$S_N=\frac{\Delta x}{3}[f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+\cdots+2f(x_{N-2})+4f(x_{N-1})+f(x_N)]\ .$$

If $I=\int_a^b f(x)\,dx$ then

$$|T_N-I|\le\frac{K_2(b-a)^3}{12N^2}\ ,\quad |M_N-I|\le\frac{K_2(b-a)^3}{24N^2}\ ,\quad |S_N-I|\le\frac{K_4(b-a)^5}{180N^4}\ .$$

The length of the curve $y=f(x)$, $a\le x\le b$ is equal to $\int_a^b\sqrt{1+(f'(x))^2}\,dx$.
The area of the surface obtained by rotating the curve $y=f(x)$, $a\le x\le b$ about the x-axis is equal to $\int_a^b 2\pi f(x)\sqrt{1+(f'(x))^2}\,dx$.
The length of the parametric curve $(x(t),y(t))$, $a\le t\le b$ equals $\int_a^b\sqrt{(x'(t))^2+(y'(t))^2}\,dt$.
If a curve is given in polar form by $r=f(\theta)$ then the area bounded by $r=f(\theta)$, $\theta=\alpha$ and $\theta=\beta$ is $\int_\alpha^\beta\frac{1}{2}r^2\,d\theta=\int_\alpha^\beta\frac{1}{2}(f(\theta))^2\,d\theta$. The length of the polar curve $r=f(\theta)$ between $\theta=\alpha$ and $\theta=\beta$ is $\int_\alpha^\beta\sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}\,d\theta=\int_\alpha^\beta\sqrt{(f(\theta))^2+(f'(\theta))^2}\,d\theta$.

Newton's Law of Cooling is given by $\frac{dT}{dt}=-k(T-T_0)$, where T is the temperature and T_0 is the ambient temperature. The balance P in an annuity is given by $\frac{dP}{dt}=r(P-N/r)$, where r is the interest rate and N is the rate of withdrawal.

The nth Taylor polynomial of $f(x)$ with center a is $T_n(x)=\displaystyle\sum_{k=0}^{n}\frac{f^{(k)}(a)}{k!}(x-a)^k$.

If $|f^{(n+1)}(u)|\le K$ for all u between a and x, then $|f(x)-T_n(x)|\le K\dfrac{|x-a|^{n+1}}{(n+1)!}$.