Math 152, Formula Sheet for the Final Exam

$$\sin(0)=0\ ;\quad \sin(\pi/6)=1/2\ ;\quad \sin(\pi/4)=\sqrt{2}/2\ ;\quad \sin(\pi/3)=\sqrt{3}/2\ ;\quad \sin(\pi/2)=1$$
$$\cos(0)=1\ ;\quad \cos(\pi/6)=\sqrt{3}/2\ ;\quad \cos(\pi/4)=\sqrt{2}/2\ ;\quad \cos(\pi/3)=1/2\ ;\quad \cos(\pi/2)=0$$

$$\cos^2 x+\sin^2 x=1\ ;\quad 1+\tan^2 x=\sec^2 x\ ;\quad 1+\cot^2 x=\csc^2 x$$
$$\sin(2x)=2\sin x\cos x\ ;\quad \cos(2x)=\cos^2 x-\sin^2 x$$
$$\cos^2 x=\frac{1}{2}\big(1+\cos(2x)\big)\ ;\quad \sin^2 x=\frac{1}{2}\big(1-\cos(2x)\big)$$
$$\sin A\cos B=\frac{1}{2}[\sin(A-B)+\sin(A+B)]\qquad \sin A\sin B=\frac{1}{2}[\cos(A-B)-\cos(A+B)]$$
$$\cos A\cos B=\frac{1}{2}[\cos(A-B)+\cos(A+B)]$$
$$\int\sec x dx=\ln|\sec x+\tan x|+C\ ;\quad \int\csc x dx=\ln|\csc x-\cot x|+C$$

If T_N, M_N, S_N are the Trapezoidal, Midpoint and Simpson's approximations, then

$$T_N=\frac{\Delta x}{2}[f(x_0)+2f(x_1)+2f(x_2)+\cdots+2f(x_{N-2})+2f(x_{N-1})+f(x_N)]\ ;$$
$$M_N=\Delta x[f(c_1)+f(c_2)+\cdots+f(c_N)] \text{ where } c_j=(x_{j-1}+x_j)/2\ ;$$
$$S_N=\frac{\Delta x}{3}[f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+\cdots+2f(x_{N-2})+4f(x_{N-1})+f(x_N)]\ .$$

Error bounds: If $I=\int_a^b f(x)dx$ then:

$$|T_N-I|\le\frac{K_2(b-a)^3}{12N^2}\ ;\quad |M_N-I|\le\frac{K_2(b-a)^3}{24N^2}\ ;\quad |S_N-I|\le\frac{K_4(b-a)^5}{180N^4}\ .$$

$$\text{arc length}: ds=\sqrt{1+\left(\frac{dy}{dx}\right)^2}dx;\quad \text{surface area}=\int_a^b(2\pi y)ds$$

For parametric curves: arc length, surface area: $ds=\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}dt.$

For polar curves: area $=\int_\alpha^\beta\frac{1}{2}r^2d\theta$; arc length: $ds=\sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}d\theta$

Newton's Law of cooling: $T'=-k(T-T_0)$.

The nth Taylor polynomial of $f(x)$ with center c is $T_n(x)=\sum_{k=0}^{n}\frac{f^{(k)}(c)}{k!}(x-c)^k$

If $|f^{(n+1)}(u)|\le K$ for all u between c and x, then $|f(x)-T_n(x)|\le K\frac{|x-c|^{n+1}}{(n+1)!}$.

$(1+x)^a=1+ax+\frac{a(a-1)}{2!}x^2+\frac{a(a-1)(a-2)}{3!}x^3+\cdots$ if $|x|<1$.