Math 152, Spring 2010, Formula Sheet for the Final Exam

$\sin(0) = 0$; $\sin(\pi/6) = 1/2$; $\sin(\pi/4) = \sqrt{2}/2$; $\sin(\pi/3) = \sqrt{3}/2$; $\sin(\pi/2) = 1$
$\cos(0) = 1$; $\cos(\pi/6) = \sqrt{3}/2$; $\cos(\pi/4) = \sqrt{2}/2$; $\cos(\pi/3) = 1/2$; $\cos(\pi/2) = 0$

$$\cos^2 x + \sin^2 x = 1 \ ; \quad 1 + \tan^2 x = \sec^2 x \ ; \quad 1 + \cot^2 x = \csc^2 x$$

$$\sin(2x) = 2 \sin x \cos x \ ; \quad \cos(2x) = \cos^2 x - \sin^2 x$$

$$\cos^2 x = \tfrac{1}{2}\big(1 + \cos(2x)\big) \ ; \quad \sin^2 x = \tfrac{1}{2}\big(1 - \cos(2x)\big)$$

$$\sin A \cos B = \tfrac{1}{2}[\sin(A - B) + \sin(A + B)]$$

$$\sin A \sin B = \tfrac{1}{2}[\cos(A - B) - \cos(A + B)]$$

$$\cos A \cos B = \tfrac{1}{2}[\cos(A - B) + \cos(A + B)]$$

$$\int \sec x \, dx = \ln|\sec x + \tan x| + C \ ; \quad \int \csc x \, dx = \ln|\csc x - \cot x| + C$$

If T_N, M_N, S_N are the Trapezoidal, Midpoint and Simpson's approximations, then

$$T_N = \frac{\Delta x}{2}[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{N-2}) + 2f(x_{N-1}) + f(x_N)] \ ;$$

$$M_N = \Delta x[f(c_1) + f(c_2) + \cdots + f(c_N)] \text{ where } c_j = (x_{j-1} + x_j)/2 \ ;$$

$$S_N = \frac{\Delta x}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{N-2}) + 4f(x_{N-1}) + f(x_N)] \ .$$

If $I = \int_a^b f(x)\, dx$ then

$$|T_N - I| \le \frac{K_2(b-a)^3}{12N^2} \ , \quad |M_N - I| \le \frac{K_2(b-a)^3}{24N^2} \ , \quad |S_N - I| \le \frac{K_4(b-a)^5}{180N^4} \ .$$

The length of the curve $y = f(x)$, $a \le x \le b$ is equal to $\int_a^b \sqrt{1 + (f'(x))^2}\, dx$.
The area of the surface obtained by rotating the curve $y = f(x)$, $a \le x \le b$ about the x-axis is equal to $\int_a^b 2\pi f(x)\sqrt{1 + (f'(x))^2}\, dx$.
The length of the parametric curve $(x(t), y(t))$, $a \le t \le b$ equals $\int_a^b \sqrt{(x'(t))^2 + (y'(t))^2}\, dt$.
If a curve is given in polar form by $r = f(\theta)$ then the area bounded by $r = f(\theta)$, $\theta = \alpha$ and $\theta = \beta$ is $\int_\alpha^\beta \frac{1}{2} r^2 \, d\theta = \int_\alpha^\beta \frac{1}{2}(f(\theta))^2\, d\theta$. The length of the polar curve $r = f(\theta)$ between $\theta = \alpha$ and $\theta = \beta$ is $\int_\alpha^\beta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta = \int_\alpha^\beta \sqrt{(f(\theta))^2 + (f'(\theta))^2}\, d\theta$.

Newton's Law of Cooling is given by $\frac{dT}{dt} = -k(T - T_0)$, where T is the temperature and T_0 is the ambient temperature. The balance P in an annuity is given by $\frac{dP}{dt} = r(P - N/r)$, where r is the interest rate and N is the rate of withdrawal.

The nth Taylor polynomial of $f(x)$ with center a is $T_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k$

If $|f^{(n+1)}(u)| \le K$ for all u between a and x, then $|f(x) - T_n(x)| \le K \frac{|x-a|^{n+1}}{(n+1)!}$

$$(1+x)^a = 1 + ax + \frac{a(a-1)}{2!}x^2 + \frac{a(a-1)(a-2)}{3!}x^3 + \cdots \quad \text{if} \quad |x| < 1$$