

Your first midterm examination is likely to contain some problems that do not resemble these review problems.

### I. Applications of Integration

(1-3) Find the volumes of the solids obtained by rotating the indicated region  $\mathcal{R}$  in the  $xy$ -plane about the specified axis:

(1) Region:  $\mathcal{R}$  is bounded by  $y = 1$ ,  $y = \ln x$  and  $x = e^2$ .

Axes: (a) the line  $y = -1$ ; (b) the line  $x = -2$ .

(2) Region:  $\mathcal{R}$  consists of all points  $(x, y)$  with  $0 \leq x \leq \pi$  and  $0 \leq y \leq \sin x$ .

Axes: (2a) the line  $y = -2$ ; (2b) the line  $x = -1$ .

(3) Region:  $\mathcal{R}$  is bounded by  $x = y(4 - y)$  and the  $y$ -axis

Axis: the  $y$ -axis.

(4) Write down the integral used to compute the work done against gravity in building a granite pyramid 500 feet high with a square base of side length 800 feet. Take the density of granite as 170 lbs per cubic foot. Explain in detail how the integral is obtained.



(5) There is a point in the interval  $[5, 7]$  where the function  $f(x) = (x^2 - 4)^{-1/2}$  takes on its average value over that interval.

(a) How do we know that? (b) Find such a point.

### II. Numerical Methods

(6) How many subintervals of  $[0, 2]$  should we use to ensure an accuracy

within  $10^{-6}$  when we approximate  $\int_0^2 4x^3 - x^4 dx$  using:

(a) the Midpoint Rule?; (b) Simpson's Rule?

(7) A certain integral  $\int_1^4 f(x) dx$  is approximated by the Trapezoidal Rule using 30 intervals, and the approximation found is 3.14286. The graph of  $f''(x)$  is shown here. Find a range of values  $[a, b]$  such that the exact value of the integral can be guaranteed to lie within that range, and explain your method. (The numbers  $a$  and  $b$  should be given to at most 3 decimal places accuracy.)

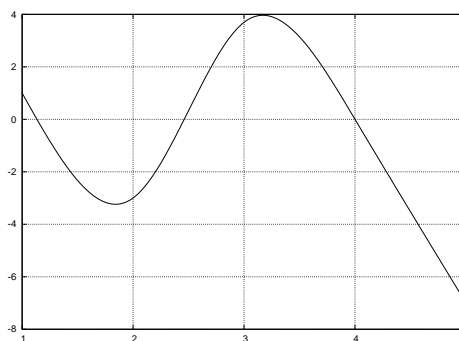


Figure 1:  $f''$

### III. Techniques of Integration

(8) Evaluate the following integrals.

$$(a) \int \sin^3 x \cos^4 x \, dx \quad (b) \int \sec^4 x \, dx$$

$$(c) \int \tan^5 x \sec^3 x \, dx \quad (d) \int \sec^3 x \, dx$$

(9) Evaluate the following integrals.

$$(a) \int x^5 (\ln x)^2 \, dx \quad (b) \int \frac{dx}{x \ln x} \quad (c) \int \cos(\sqrt{x}) \, dx$$

$$(d) \int x^2 \tan^{-1} x \, dx \quad (e) \int x^{-2} \sin^{-1} x \, dx \quad (f) \int e^{\sqrt{x}} \, dx$$

$$(g) \int_{-\pi/2}^{\pi/2} \frac{\cos(x)}{1 + e^x} \, dx$$

(10) Evaluate the following integrals.

$$(a) \int \frac{dx}{(25 + x^2)^2} \quad (b) \int \frac{x \, dx}{(x^2 + 36)(x + 1)} \quad (c) \int \frac{dx}{\sqrt{2x - x^2}}$$

$$(d) \int \frac{x^2 - x + 4}{(x - 5)(x + 3)^2} \, dx \quad (e) \int \frac{x^2 \, dx}{(16 - x^2)^{3/2}} \quad (f) \int \frac{dx}{x^2 + 4x + 9}$$

(11) Evaluate  $\int \sin(\ln x) \, dx$  using two integrations by parts. Would another method work?