

Math 152, Spring 2009, Review Problems for Exam 1

Your first exam is likely to have problems that do not resemble these review problems.

(1) Consider the region \mathcal{R} in the xy -plane which is bounded by $y = 1$, $y = \ln x$ and $x = e^2$. Find the volume of the solid obtained by rotating \mathcal{R} about the line $y = -1$. Find the volume of the solid obtained by rotating \mathcal{R} about the line $x = -2$.

(2) Consider the region \mathcal{R} in the xy -plane consisting of all points (x, y) with $0 \leq x \leq \pi$ and $0 \leq y \leq \sin x$. Find the volume of the solid obtained by rotating \mathcal{R} about the line $y = -2$. Find the volume of the solid obtained by rotating \mathcal{R} about the line $x = -1$.

(3) Consider the region \mathcal{R} in the xy -plane which is bounded by $x = y(4 - y)$ and the y -axis. Find the volume of the solid obtained by rotating \mathcal{R} about the y -axis.

(4) A 100 foot tall pyramid has a square base with sides of length 150 feet. Find the volume of this pyramid.

(5) How many subintervals of $[0, 2]$ should we use to get an accuracy better than 10^{-6} when we approximate $\int_0^2 4x^3 - x^4 dx$ with the Midpoint Rule?

(6) Find the Simpson Rule approximation and the Trapezoidal Rule approximation to $\int_1^3 \frac{dx}{x}$ using 4 subintervals of $[1, 3]$.

(7) On a flat beach with no incline, we build a 3 foot tall sand castle such that every horizontal cross-section is a region bounded by two concentric circles with radii 2 feet and 1.5 feet. Find the work done against gravity if the weight density of the beach sand is 200 pounds per cubic foot. For simplicity, assume that all of the sand was scooped up from ground level.

(8) Evaluate $\int \sec x dx$, $\int \csc x dx$, $\int \sec^3 x dx$, $\int \frac{x dx}{(x^2 + 36)(x + 1)}$.

(9) Find the average value of the function $f(x) = (x^2 - 4)^{-1/2}$ on the interval $[5, 7]$.

(10) Evaluate $\int \sin^3 x \cos^4 x dx$, $\int \tan^5 x \sec^3 x dx$, $\int \sec^4 x dx$, $\int \frac{dx}{(25 + x^2)^2}$.

(11) Evaluate $\int \sin(\ln x) dx$ and $\int \sin(7x) \sin(5x) dx$ using two integrations by parts for each integral.

(12) Evaluate $\int x^5 \ln x dx$, $\int x^5 (\ln x)^2 dx$, $\int \frac{x^2 - x + 4}{(x - 5)(x + 3)^2} dx$, $\int \frac{x^2 dx}{(16 - x^2)^{3/2}}$.

(13) Evaluate $\int \frac{dx}{\sqrt{2x - x^2}}$, $\int \cos(\sqrt{x}) dx$, $\int e^{\sqrt{x}} dx$, $\int \cos^4 x dx$.

(14) Evaluate $\int x^2 \tan^{-1} x dx$, $\int x^{-2} \sin^{-1} x dx$, $\int \frac{dx}{x^2 + 4x + 9}$.