Math 152, Spring 2011, Review Problems for Midterm Exam 2

Your second midterm examination is likely to contain some problems that do not resemble these review problems.

(1) Evaluate the following improper integrals:

$$\int_5^\infty \frac{dx}{(x-3)(x-4)} \qquad \int_0^1 \ln x\,dx \qquad \int_0^\infty \frac{x^3\,dx}{e^x} \qquad \int_{-\infty}^\infty \frac{dx}{9+x^2}$$

(2) Show that one of these improper integrals converges, and that one them diverges:

$$\int_7^\infty \frac{dx}{x-|\cos(x)|} \qquad \int_5^\infty \frac{dx}{e^{x^2}}$$

(3) Find the length of the cardioid $r = 1 - \cos\theta$, $0 \le \theta \le 2\pi$.

(4) Find the area inside the cardioid $r = 1 - \cos\theta$, $0 \le \theta \le 2\pi$.

(5) Find the length of the curve $y = (x+2)^{3/2}$, $0 \le x \le 1$.

(6) Use calculus to find the surface area of a sphere of radius R.

(7) Find the center and radius of the circle $r = \sin\theta$, $0 \le \theta \le \pi$.

(8) Find a parametrization $x = f(t)$, $y = g(t)$ of the ellipse $9x^2 + 16y^2 = 36$.

(9) Find the length of the curve given parametrically by $x = t^2$, $y = t^3$, $1 \le t \le 2$.

(10) Solve the following two initial value problems:
(a) $\dfrac{dx}{dt} = \tan x$, $x(0) = \pi/6$.
(b) $(4+x^3)^{1/2}\dfrac{dy}{dx} = (xy)^2$, $y(0) = -1$.

(11) A room has an ambient temperature of 25°C. The room contains a cup of coffee that starts out at a temperature of 50°C and cools down to a temperature of 30°C after sitting in the room for one hour. How long did it take for the coffee to go from 50°C to 40°C?

(12) An annuity is supposed to run out of money 30 years after it was initially set up. The interest rate is 5 percent and money is withdrawn continuously at a rate of 20,000 dollars per year. What was the initial balance in the account?

(13) Find the Taylor polynomial $T_3(x)$ of the function $f(x) = \frac{1}{x}$ with center at $a = 1$. Find an estimate for $|f(3/2) - T_3(3/2)|$ using the Error Bound.

(14) Evaluate the following limits:

$$\lim_{n\to\infty} \frac{\sin n}{n} \qquad \lim_{n\to\infty} (3n)^{1/n} \qquad \lim_{n\to\infty} \left(1 - \frac{5}{n}\right)^n$$

$$\lim_{n\to\infty} n\big(\sin(1/n)\big) \qquad \lim_{n\to\infty} n^2\big(1 - \cos(1/n)\big)$$

(15) Write the repeating decimal $5.273273273\ldots$ in the form p/q, where p and q are natural numbers.

(16) Evaluate each of the following sums. Your answers must be simple numbers.

$$\sum_{n=3}^{\infty} \frac{2^n}{3^{n+1}} \qquad \sum_{n=4}^{\infty} \frac{1}{n(n-1)}$$

(17) For each series below, determine whether it converges or diverges. Explain your reasons.

$$\sum_{n=5}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}} \qquad \sum_{n=5}^{\infty} \frac{1}{n(\ln n)} \qquad \sum_{n=5}^{\infty} \frac{1}{n(\ln n)^{3/2}}$$

$$\sum_{n=4}^{\infty} \left(1 - \frac{5}{n}\right)^n \qquad \sum_{n=4}^{\infty} \sin(1/n) \qquad \sum_{n=4}^{\infty} \big(1 - \cos(1/n)\big)$$

$$\sum_{n=2}^{\infty} \frac{2^n}{3^n + 1} \qquad \sum_{n=2}^{\infty} \frac{n^2}{n^4 - n^3 - 4}$$

(18) Estimate the error involved in taking the sum of the first 10 terms of each of the following series as an approximation to the full sum.

$$\sum_{n=1}^{\infty} \frac{1}{n^2} \qquad \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$$

(19) Define *absolute convergence*, and give an example of a series that converges conditionally, but does not converge absolutely.

(20) An infinite sequence is defined by $a_1 = 1$ and $a_{n+1} = \sqrt{12 + a_n}$ for $n = 1, 2, 3, \ldots$. Assume that the sequence converges. Find $\lim_{n\to\infty} a_n$.