To review for the final exam, use your homework assignments, your midterms, and the two previous review sheets supplied by the department. This last sheet contains 13 additional review problems relating to material covered recently.

The final examination is a common examination, set by the department, and graded by the department. It covers the work of the entire term. Calculators and notes are not permitted. A formula sheet will be supplied.

The final examination for Math 152 will be given on Monday, December 19, 2011 from 4:00 pm to 7:00 pm. Your final exam schedule can be found at http://finalexams.rutgers.edu/.

Additional Review Problems on Sequences and Series

1. Interval of Convergence
Find the precise interval of convergence for each of the series

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(x-3)^n}{(n+1)2^n}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(x+3)^n}{\sqrt{n+1}}$

Check the endpoints, and specify whether the convergence is conditional or absolute.

2. Interval of Convergence
If there is a power series with the specified interval of convergence, give an example. If not, explain why not.

(a) $(-2, 1)$ (b) $[0, \infty)$ (c) $(-1, 1]$

3. Tests for Convergence and Divergence
State which tests are used in each case to determine whether the series converges, and what the conclusion is.

(a) $\displaystyle\sum_{n=7}^{\infty} \frac{(-1)^n(n+3)}{n+6}$ (b) $\displaystyle\sum_{n=2}^{\infty} \sqrt{\frac{n^2+1}{n^5-n^4-2}}$ (c) $\displaystyle\sum_{n=1}^{\infty} \sin(1/n)$

(d) $\displaystyle\sum_{n=3}^{\infty} \frac{\sin^2(e^n)}{n^{4/3}}$ (e) $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{3^n n!}$ (f) $\displaystyle\sum_{n=3}^{\infty} \frac{3n^2-n-1}{5n^3+n^2+5}$

(g) $\displaystyle\sum_{n=1}^{\infty} \sin(1/n^2)$ (h) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n+n}{4^n-3^n}$ (i) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^4}$

(j) $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)!n!}{(3n)!}$ (k) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$ (l) $\displaystyle\sum_{n=3}^{\infty} \frac{\cos(n^3)}{n(\ln n)^2}$

(m) $\displaystyle\sum_{n=2}^{\infty} \frac{7^n n^7}{2^{2n-3}}$ (n) $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^2(\pi)^n}{(2n)!}$ (o) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+3}\sqrt{n+3}$

4. Algebraic manipulations with Power Series.

(a) The "hyperbolic cosine" $(\cosh(x))$ is defined by $\cosh(x) = \dfrac{e^x + e^{-x}}{2}$. Find its Maclaurin expansion. How is this series related to the Maclaurin expansion of $\cos(x)$?

(b) Calculate the first 3 nonzero terms of the Maclaurin series for $\tan x$.

(c) Calculate the first 5 nonzero terms of the Maclaurin series for $x^3 e^{-x^2}$.

5. Evaluation of power series

Let $f(x)$ be defined by $f(x) = \displaystyle\sum_{n=0}^{\infty} \dfrac{2^n x^n}{n!}$

(a) Show that $f'(x) = 2f(x)$

(b) Find an explicit formula for $f(x)$.

6. The Binomial Series Find the first 4 nonzero terms of the Maclaurin expansion of $f(x) = \sqrt{4 + x^2}$.

7. Convergence of Power Series

Use the Maclaurin expansion for e^x to show that

$$e = \sum_{n=0}^{\infty} \frac{1}{n!}$$

What theorems about series are involved in justifying this claim?

8. Evaluating Power Series

Find the Taylor expansion of $\dfrac{1}{(1-x)^2}$ by the following three methods.

(a) Apply the Binomial Theorem.

(b) Square the series for $\dfrac{1}{(1-x)}$.

(c) Differentiate the series for $\dfrac{1}{(1-x)}$.

Rank the methods according to your preference (for this particular case).

9. Summation Formulas Calculate $\displaystyle\sum_{n=1}^{\infty} \dfrac{n}{2^n}$ exactly:

(a) Obtain a simple formula for the sum $\displaystyle\sum_{n=1}^{\infty} nx^n$.

(b) Check that $x = 1/2$ is in the interval of convergence, and then take $x = 1/2$.

10. Numerical Methods for Integration: Power Series

(a) Use the Maclaurin expansion of degree 3 for $\sin x$ to estimate $\displaystyle\int_0^{1/2} \frac{\sin(\sqrt{x})}{\sqrt{x}}\, dx$.

(b) Give an estimate for the resulting error.

(c) How many (non-zero) terms of the expansion would be needed to meet an error tolerance of 10^{-6}?

11. Limits

Find $\displaystyle\lim_{n\to\infty}\left[\frac{\sin\left(\frac{1}{n}\right) - \left(\frac{1}{n}\right)}{\left(e^{\frac{1}{n}} - 1\right)\left(1 - \cos\left(\frac{1}{n}\right)\right)}\right]$ using the Maclaurin expansions for $\sin$, $\cos$, and e^x.

12. Computation of $\sqrt{5}$ by Newton's Method: Convergence.

In the computation of $\sqrt{5}$ by Newton's Method in your text (§4.8, p. 280), the following sequence is defined:

$$a_0 = 2;\quad a_{n+1} = \frac{a_n + (5/a_n)}{2}.$$

Show that this sequence converges to $\sqrt{5}$, as follows:

(a) Show that $a_n \geq \sqrt{5}$ for $n \geq 1$

Hint: What is the minimum value of $x + \dfrac{5}{x}$ for $x > 0$?

(b) Show that the sequence (a_n) is decreasing for $n \geq 1$.

(c) Deduce that the sequence a_n has a limit L.

(d) Show that $L = \sqrt{5}$.

13. p-series with $p = 2$

The mathematician Leonhard Euler showed in 1734 that the sum $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$

is equal to $\dfrac{\pi^2}{6}$ (this took him 4 years). Using this fact, calculate the following related sums:

(a) $A = \displaystyle\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2}$ (b) $B = \displaystyle\sum_{n=1}^{\infty} \frac{1}{(2n)^2}$ (c) $C = \displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$

Hint: The easiest is (b).

How could each of these series be used to calculate π?