Math 152, Spring 2009, Review Problems for the Final Exam

Your final exam is likely to have problems that do not resemble these review problems. You should also study the review sheets for the first and second exams. This final exam review sheet emphasizes the topics that were covered after the second exam. However, the final exam will NOT place great emphasis on sections 10.6, 10.7, 11.1-11.4 of the textbook at the expense of earlier sections.

(1) (a) Find the interval of convergence of $\sum_{n=1}^{\infty} \frac{(x-3)^n}{(n+1)2^n}$. (b) Find the interval of convergence of $\sum_{n=1}^{\infty} \frac{(-1)^n (x-3)^n}{\sqrt{n+1}}$.

(2) (a) Find a power series such that its interval of convergence is exactly $(-1, 3)$. (b) Find a power series such that its interval of convergence is exactly $[-1, 3]$.

(3) Let $f(x)$ be defined by $f(x) = \sum_{n=0}^{\infty} \frac{2^n x^n}{n!}$. Show that $f(x)$ is a solution of the differential equation $f'(x) = 2f(x)$ with initial condition $f(0) = 1$. Use this to conclude $e^{2x} = \sum_{n=0}^{\infty} \frac{2^n x^n}{n!}$.

(4) Find the first 3 nonzero terms of the Maclaurin series of $\tan x$ using long division and the Maclaurin series of $\sin x$ and $\cos x$.

(5) Express $\int_0^2 \frac{\sin x}{x}\, dx$ as the sum of an infinite series with rational terms. Hint: Use the Maclaurin series of $\sin x$.

(6) Find the first 4 nonzero terms of the Maclaurin series of $f(x) = \sqrt{4 + x^2}$. Hint: Use a binomial series and the fact $4 + x^2 = 4(1 + (x^2/4))$.

(7) Obtain a simple formula for the sum $\sum_{n=1}^{\infty} n x^n$ using the product of the Maclaurin series of $\frac{1}{1-x}$ and the Maclaurin series of $\frac{x}{1-x}$.

(8) Find a parametrization $x = f(t)$, $y = g(t)$ of the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$.

(9) Find the point (x, y) where the parametric curve $x = t^2 - 9$, $y = t^3 - t$ crosses itself. Find the equations of both tangent lines at that point (x, y).

SEE NEXT PAGE

(10) Consider the parametric curve $x = t^3 - 27t + 5$, $y = t^3 - 12t + 3$. Find the points (x, y) where the curve has horizontal tangents, and the points (x, y) where the curve has vertical tangents.

(11) Find the length of the curve given parametrically by $x = t^2$, $y = t^3$, $1 \le t \le 2$.

(12) Show that the polar equation $r = 8\cos\theta$ gives us a circle in the xy-plane.

(13) Find the area of the region in the xy-plane which is bounded by $r = 8\cos\theta$ and the rays $\theta = \pi/4$, $\theta = \pi/2$.

(14) Find the length of the cardioid $r = 1 - \cos\theta$, $0 \le \theta \le 2\pi$. Hint: $1 - \cos(2t) = 2\sin^2(t)$.

(15) Evaluate $\displaystyle\int \frac{x^2 + x + 2}{(x^2 - 4)(x + 2)}\,dx$ and $\displaystyle\int x^2\sqrt{16 - x^2}\,dx$.

(16) Evaluate $\displaystyle\int_{-\infty}^{\infty} \frac{dx}{x^2 + 4x + 8}$ and $\displaystyle\int_{-1}^{1} \frac{dx}{\sqrt{|x|}}$.

(17) Evaluate $\displaystyle\int (\ln x)^2\,dx$, $\displaystyle\int \sin^{-1} x\,dx$ and $\displaystyle\int \tan^{-1} x\,dx$.

(18) Find the length of the curve $y = x^2$, $0 \le x \le 1$.

(19) Solve the differential equation $\dfrac{dy}{dx} = \dfrac{y^3}{x^3}$ with initial condition $y(1) = 2$.

(20) Find the volume of a right circular cone of height H and radius R in two different ways: Using the method of disks and using the method of shells.

(21) Find $\displaystyle\lim_{n\to\infty} \sqrt{n}\big(\sqrt{n+5} - \sqrt{n}\big)$.

(22) Assume that the sequence a_n is defined by $a_1 = 10$, $a_{n+1} = a_n + \dfrac{1}{a_n}$. Show that a_n is not bounded. Hint: Assume it is bounded. Show it is increasing. What conclusion can you draw?

(23) Use appropriate tests to determine convergence or divergence for each of the following series:

$$\sum_{n=1}^{\infty} \frac{n^3}{n^5 + 4n + 2}\ , \quad \sum_{n=1}^{\infty} \sin(1/n)\ , \quad \sum_{n=1}^{\infty} \sin(1/n^2)\ , \quad \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^4}$$

$$\sum_{n=1}^{\infty} \frac{(2n)!n!}{(3n)!}\ , \quad \sum_{n=1}^{\infty} \frac{(-1)^{n+3}}{\sqrt{n+3}}\ , \quad \sum_{n=1}^{\infty} (-1)^{n+3}\sqrt{n+3}$$

(24) Find N such that the Midpoint Rule with N subintervals approximates $\displaystyle\int_0^4 e^{-x^2}\,dx$ with accuracy better than 0.000001.