Intro to Mathematical Reasoning (Math 300)

Supplement 4. **Using letters in mathematical writing** [1]

As we all know, letters are used in proofs and other mathematical writing to represent mathematical objects, such as numbers, matrices, sets and functions. The use of letters for this purpose is governed by certain rules. Failure to follow these rules can cause great confusion, and result in serious errors in mathematical reasoning. This supplement explains the guidelines for using letters that will help you avoid these errors.

Letters in mathematical writing fall into two main types. We refer to these types as:

- *object names*
- *dummy variables.*

Roughly speaking, an object name is a letter that represents a particular (possibly unknown) mathematical object that we want to refer to over two or more sentences. Once we designate a letter as an object name representing a particular object, that letter can only be used to refer to that same object. This will be explained and clarified below.

A dummy variable is a letter that is used to make it easier to express a single sentence. When we use a letter as a dummy variable, that letter has a clear meaning in that sentence, but has no meaning outside the sentence. Dummy variables are essentially the same as what was called a bound variable in supplement 2. They include letters that are quantified in a sentence, such as the letter x in the sentence "For all real numbers $x \geq 1$, $x^2 \geq x$".

The terms "object name" and "dummy variable" are not standard mathematical terms.

This supplement, like supplement 2, is rather abstract and many students may find it hard to understand on first reading. During your first reading, your goal is to develop a basic understanding for the differences between object names and dummy variables, and for the rules governing their usage. You should then keep this supplement as a handy reference when you are writing proofs, and follow the rules. Be especially careful to keep straight in your mind which letters you are using are object names and which are dummy variables. This supplement will be especially helpful in understandinge why a proof is unacceptable. The grader may indicate that you have misused a letter in the proof; if so, then the supplement should contain an explanation of the rule that you violated.

The rules presented here are stricter than those normally followed by experienced mathematicians. In many activities, beginners learn rules that experts don't always follow. If you take a rock climbing class, then among the first and most important things taught are safety rules. These rules serve a very important purpose: keeping the beginning rock climber from bodily harm. A beginner may see an experienced climber violate some of the rules. Why, wonders the beginner, do I need to follow all of these rules when these experts don't?

The reason is this: the expert has judgement developed out of long experience which enables him to recognize situations where he can safely ignore a safety rule. The beginner does not have this judgement, and may not be able to distinguish between a situation where a particular safety precaution is unnecessary, and a situation where it is critically important. So the beginning climber is advised to follow all of the rules all of the time until he has more experience.

The same is true of writing proofs. The rules here, as is the case with all of the rules of mathematical proof, are intended to protect you from ever claiming that you have proved a statement that is actually false (the mathematical equivalent of falling to your death). For experts, some of these rules are sometimes unnecessary, but it requires judgement to know when you can safely cheat on the rules. As beginners, most students lack this judgement and should therefore follow the rules.

You will see that even the textbook does not strictly follow these rules. That does not mean that the proofs in the textbook are wrong, only that the author has chosen to be more relaxed about the rules in situations where the rules are not crucial. Nevertheless, for the reason explained above, it is recommended that students use the rules that will be described, until they gain the experience necessary to safely relax them.

1 Object Names

Most fiction writing is concerned with telling a story involving certain human characters. In writing a novel, the author introduces each character and (normally) gives a different name to each character so that you and the reader can tell them apart. Also, the author describes certain characteristics of the character (for example "Jose is a 30 year old unmarried policeman, whose parents are immigrants from Mexico.") The characteristics of the character provide information about them that is often crucial in understanding the story.

There are important similarities and differences between fiction and mathematical exposition. A mathematical exposition (such as a proof) can also be thought of as telling a story involving certain mathematical "characters". The characters in a mathematical exposition are specific mathematical objects, such as "an even number", "a set of integers with 10 elements", "a 3 by 3 matrix with real number entries". In order to keep the different objects in the exposition straight, we must give each one a name. In mathematical writing the names we give to objects are usually letters of the English alphabet or Greek letters. We can also use letters with subscripts, such as "x_3" or "y_{17} or letters with additional markings such as x' or $\hat{x}$. We will refer to any such name as a "letter", even though it might involve more than just a single letter.

In both fiction and mathematical writing, there are various rules which one learns in order to write well. In fiction, these rules are very flexible. For example, an author may choose to call two characters "Paul" even though this is generally to be avoided. In mathematical writing, the rules are much more rigid. There are four basic rules for the use of object names:

- Object Introduction (OI) rule.
- Complete specification (CS) rule.
- Existence first (EF) rule.
- Unique names (UN) rule.

We will discuss these rules in detail below.

1.1 The "Object Introduction" (OI) rule.

This rule says that whenever you use a letter to represent an object in a mathematical proof or discussion, the *very first time* that you mention that letter you must give a *proper introduction* that tells the reader what that letter will represent in your discussion. Here are some examples:

> Let m be a positive even integer.
>
> Let a be a real number that satisfies $a^3 \geq a^2 + 10$.
>
> Let S be an arbitrary set of real numbers with 10 members.

Each of these sentences introduces a letter to stand for a mathematical object. When we introduce the letter "x" to stand for an object the general form is: "Let x be a ..." followed by a list of conditions. That list of conditions is called the *specification* of the object. In the first example the specification is "a positive even integer".

We saw this earlier, when we learned about the arbitrary value method for proving universal statements. If you are proving a statement of the form "$\forall x$ satisfying $H(x)$, $C(x)$ must hold", then such a proof starts with a sentence like. "Let a be an arbitrary object that satisfies $H(a)$." In these notes, we use the word "arbitrary" to indicate that we are using the arbitrary value method (this is common, but not standard, practice among mathematicians.)

Here the letter a is introduced to start the arbitrary value method and the goal is to demonstrate that $C(a)$ must hold. In general, when we use the arbitrary value method to prove a universal statement U and we introduce an object (or objects) corresponding to the universally quantified variable (or variables) then the introduction specifies that they satisfy the hypothesis of the universal statement. We refer to these as *hypothesized objects*.

Often, it is useful to introduce other objects in your proofs. Such objects are referred to here as *deduced objects*.

Example. In supplement 3 we proved "For all real numbers x, if $x < 2$ then there is a real number z such that $x < z$ and $z < 2$." The proof started:

> Let a be an arbitrary real number satisfying $a < 2$. We will show that there exists a real number z such that $a < z$ and $z < 2$. Let $b = (a + 2)/2$.

Here a is the name of hypothesized object name, while b is the name of deduced object. As we will see below, z is not an object name, but is a dummy variable.

This example shows something else; when you introduce a letter to stand for an object, you may refer to a previously introduced letter. Here the object b is introduced by referring to a. *However, if a was not previously introduced, then the introduction "Let $b = (a + 2)/2$" is not allowed!*

In general, once you introduce an object name, you may refer to that object in your future discussion and treat it as though it is a specific object.

What exactly does it mean when a mathematician writes: "Let x be an object satisfying $S(x)$"? Here is what it means:

> In this discussion x stands for a mathematical object. The object x is known to satisfy $S(x)$, but nothing else is known about x. If you, the reader, select any value for x that satisfies $S(x)$, and substitute it for x throughout this proof, then the resulting proof should still be correct.

Other ways to introduce object names

Mathematicians sometimes use other language to introduce object names such as "Suppose a is an integer"; "Consider an integer a"; "Assume a is an integer".

When you read mathematics, you may see these and other introductions. For now, until you become accustomed to proper introduction of object names, it is strongly suggested that you not use these and that you use the above construction with "Let ...".

There is another form of introduction that can be used in combination with an existential quantifier. This will discussed below when we discuss the existence first (EF) rule.

Implied object name introductions

There are situations where writers of mathematics commonly omit the introduction of an object name, and rely on the reader to mentally fill it in. This is permitted, provided that it is done appropriately. However, many students lack the experience to know when it is safe to do this and when it is not, and so students are advised against using these shortcuts until they gain more experience.

There are two common situations where this is done. One of these will be explained in the section about the existence first (EF) rule. We now describe the other one, which is often used in our textbook.

When proving a statement of the form "For all x satisfying $H(x)$, x must satisfy $C(x)$." we have seen that a proof using the arbitrary value method starts something like "Let a be an object satisfying $H(a)$."

Many writers of mathematics do something different. Because the statement to be proved uses the letter x, they write their proof as though x has been introduced satisfying $H(x)$. For example given the statement "For all real numbers x, y, z, $x^2 + y^2 + z^2 \geq xy + xz + yz$.

> Proof. We will show that $x^2+y^2+z^2-xy-xz-yz \geq 0$. We have that $x^2+y^2+z^2-xy-xz-yz = \frac{1}{2}\left[(x^2-2xy+y^2)+(x^2-2xz+z^2)+(y^2-2yz+z^2)\right]$ and this is equal to $\frac{1}{2}\left[(x-y)^2+(x-z)^2+(y-z)^2\right]$. Since the square of any integer is nonnegative, the expression inside the brackets is nonnegative and so $x^2+y^2+z^2-xy-xz-yz \geq 0$. Therefore $x^2+y^2+z^2 \geq xy+xz+yz$.

Here the writer begins by referring to x, y, z as though they are object names that have already been introduced. But they have not been. Here is the recommended way for students to write this proof:

> Proof. Let a, b, c be arbitrary real numbers. We will show that $a^2+b^2+c^2-ab-ac-bc \geq 0$. We have that $a^2+b^2+c^2-ab-ac-bc = \frac{1}{2}\left[(a^2-2ab+b^2)+(a^2-2ac+c^2)+(b^2-2bc+c^2)\right]$ and this is equal to $\frac{1}{2}\left[(a-b)^2+(a-c)^2+(b-c)^2\right]$. Since the square of any integer is nonnegative, the expression inside the brackets is nonnegative and so $a^2+b^2+c^2-ab-ac-bc \geq 0$. Therefore $a^2+b^2+c^2 \geq ab+ac+bc$.
>
> Since a, b, c are arbitrary real numbers, we conclude that for all real numbers x, y, z, $x^2+y^2+z^2 \geq xy+xz+yz$.

In the first version, the letters x, y, z that appear in the initial statement are dummy variables. They have not been introduced, and so technically, can not be referred to in later sentences. The experienced mathematician reading or writing the first version of the proof recognizes that, since we are proving a universal statement, there is an invisible introduction missing "Let x, y, z be arbitrary real numbers", and mentally fills this in.

The second proof not only includes the introduction explicitly, but uses three different letters in place of x, y and z. It also concludes with a summary statement explaining exactly what was proved.

Why are students recommended to use the second version rather than the first one? Many students have a lot of trouble keeping track of which letters are object names and which are dummy variables. Confusing dummy variables and object names can be disastrous for a proof. The explicit introduction of object names serves as an important reminder that the rules of object names must be followed. Using different letters for the object names than were used for the dummy variables in the theorem statement helps further to prevent this confusion. The final summary sentence helps to make it clear that the goal of the proof has been accomplished.

The Life span of an object

When you introduce a letter, say x, as an object name, that object has a well defined *life span* (note: the term "life span" is not a standard mathematical term!) During the life span of the object, the letter x stands for a particular object (that is usually not completely specified). Once the life span is over, the letter x "dies" and it no longer stands for an object. The letter x may then be used for another purpose, for example it can be reintroduced (or, if you prefer, reborn) to stand for another object.

Usually the life span of the object is the entire proof. However, sometimes the proof you are doing breaks up into two or more separate *subproofs* (this occurs, for example, in *proof by cases* which will be discussed in detail in a later supplement). If you introduce x inside of a subproof, then the life span of x is only for the length of the subproof. Once the subproof is over, x's current life is over. *You may not refer to x outside of the subproof, unless you reintroduce it.* We will see an example in the supplement on proof by cases.

Another similar situation is this: When proving a theorem, you may need to use a fact that you have not yet proved. What you need to do then is interrupt your proof, state the fact and then prove the fact. You can then continue the proof of your theorem using that fact. A fact that you prove in order to use it inside another proof is often called a *lemma*. Any objects that you introduce inside the proof of the lemma dies when the proof ends.

Reserved letters

In some cases, there are quantities that are so useful that mathematicians have agreed that certain letters will stand for them. For example the letter e is the common abbreviation for the number that is equal to $\lim_{n\to\infty}(1+1/n)^n$, and π is the common abbreviation for the number that is the ratio of the circumference of a circle to its diameter. Generally, you should avoid using these letters to mean other things, if there is any chance of confusion.

1.2 The "Complete specification" (CS) rule

When you introduce an object and give it a name, you must completely specify all the properties that you are assuming about the object. Once you complete the sentence introducing the named object, you can not in later sentence add more conditions.

For example suppose you want to introduce n to stand for a prime number that is odd. It is *incorrect* to write:

> Let n be a prime number. n is odd.

The first sentence introduces n and says n is a prime number. The introduction of n ends with the first sentence. As described above, we now require all later sentences to be true for any choice of n satisfying the specification. Since 2 is prime it satisfies the specification, but if we substitute 2 for n, the second sentence becomes false.

The correct introduction would be: "Let n be an odd prime number" or "Let n be a prime number that is odd."

Sometimes, during a proof, you may want to add some additional properties to the object that were not part of the initial specification. For example, you might have made the introduction "Let x be an integer." Later in your proof, you have a different argument depending on whether x is even, or x is odd. In this situation, you use proof by cases and separately consider the possibility that x is even and that x is odd. We saw an example of proof by cases in supplement 3, and will see more in a later supplement.

1.3 The "Existence First" (EF) rule

When you introduce a letter to represent a certain object, then you need to already have shown (using known facts, or assumptions that were made according according to the rules of making assumptions) that there is at least one object of the type you are introducing.

To see why we need this rule, consider the ridiculous statement "For all real numbers r and s, $r < s$." Here is a proposed proof.

> Let a and b be arbitrary real numbers. Let S be the set of all real numbers that are greater than a. Let T be the set of real numbers that are less than b. Let x be a real number that belongs to both S and T. Then since $x \in S$, we have $a < x$ and since $x \in B$, we have $x < b$. Since $a < x$ and $x < b$ we have $a < b$.
>
> Since a and b were arbitrary, we conclude that for all all real numbers r and s, $r < s$.

The proof must be faulty, since it is proving a false statement. We can uncover the error using trial values. Suppose we take $a = 2$ and $b = 1$. Then S is the set of real numbers that are greater than 2 and T is the set of real numbers that are less than 1. When we introduce "Let x be a real number that belongs to both A and B", it is impossible to carry out those instructions since there is no real number that belongs to both A and B.

The EF rule protects against this kind of error. Under the EF rule, before you can introduce x to be a real number that belongs to both A and B, you would first need to prove, "There exists a real number z such that z belongs to both A and B." Only after proving this could you introduce x.

In general, if you want to say:

> Let x stand for an object that satisfies the condition $P(x)$,

then you need to first prove: "there is at least one object z that satisfies $P(z)$."

Example. Suppose n has been introduced to be an even integer and you want to introduce k to be an integer satisfying $n = 2k - 4$. Assume that we previously proved that the sum of even numbers is even. Then you might write:

> Since n is even and 4 is even, and the sum of even numbers is even, we have that $n + 4$ is even. Since $n + 4$ is even then, by the definition of even, there is an integer k such that $2k = n + 4$ which implies that $n = 2k - 4$.

Here we have established "There exists an integer k such that $n = 2k - 4$."

However, we now have to be careful: while we have proved the existence of an integer k such that $n = 2k - 4$, we have not properly introduced k. In the last sentence, the letter k appears with an existential quantifier and is therefore a *dummy variable*, as explained in the next section. You can not treat k as though it were introduced.

In a previous homework problem, you were asked to uncover the error in a faulty proof that "For all integers x, y, z, if z is a divisor of x and z is a divisor of y then $2z$ is a divisor of $x + y$." The error in the proof can be viewed as a failure to properly distinguish between dummy variables and object names.

Here is a correct way to introduce k.

> Since n is even and 4 is even, and the sum of even numbers is even, we have that $n + 4$ is even. Since $n + 4$ is even, by the definition of even, there is an integer w such that $2w = n + 4$ and thus $n = 2w - 4$. Let k be an integer such that $n = 2k - 4$.

Here we use w as the dummy variable when we prove the existence of the object we want to create, and then we introduce it as k.

This wording seems rather awkward and repetitive. For this reason, experienced mathematicians often cheat here. They will write simply "There exists an object z that satisfies $A(z)$", and then proceed as though z had been introduced. (This is another example of "implied object name introductions") As just explained, this is not proper usage, and can result in faulty proofs. It is an example of a "risky shortcut" that an experienced mathematician takes because she knows how to abuse the rules safely. An experienced mathematician will (rarely) make the kind of mistake that appears in the faulty proof from the homework mentioned above, but such mistakes are common for beginners. Therefore, until you learn when it is safe to take such shortcuts and when it is not, you shouldn't do it.

Nevertheless, there is an acceptable short cut that you can use. Instead of writing:

> There exists an object z that satisfies $A(z)$. Let k be an object satisfying $A(k)$,

you may combine these two sentences into one and write:

> There exists an object, which we will call k, that satisfies $A(k)$.

By including the phrase "which we will call" the writer alerts the reader that k is now the name of an object which may be referred to later. If the phrase "which we will call" is omitted, then, in these notes, k is a dummy variable and can not be referred to later.

There are two apparent exceptions to the EF rule (which are not really exceptions):

- (Objects whose existence is obvious.) If it *clearly* follows from basic facts that we have assumed in the course, or that we have proved, then you don't have to do this. For example, if you want to introduce a n to be an integer greater than 10, then you do not have to first prove that such an integer exists. *However, when in doubt, you should prove that the needed object exists.*

- (Hypothesized objects) Recall that earlier objects come in two types: hypothesized objects and deduced objects. Hypothesized objects are those that are introduced at the start of the arbitrary value method. For hypothesized objects you need not first prove existence. For example, if you are proving a universal statement "For all x satisfying $H(x)$, x must satisfy $C(x)$" using the arbitrary value method, you may write introduce: "Let a be an arbitrary object satisfying $H(x)$", without first proving that there is an object satisfying $H(x)$.

  (Here is an explanation for this exception, which you can safely skip the first time you read this supplement. When you introduce a to satisfy $H(a)$ you are assuming that "There exists x satisfying $H(x)$" is true, even though it might be false. Suppose you succeed in showing that $C(a)$ is true. We then want to conclude "For all x satisfying $C(x)$, x must satisfy $H(x)$". This is a correct conclusion, even though our initial assumption "There exists x satisfying $H(x)$" might have been false. Here's why: if the assumption "There is at least object x satisfying $H(x)$" is false, then the statement to be proved is *vacuously true*, and thus we are safe to reach the conclusion "For all x satisfying $C(x)$, x must satisfy $H(x)$".)

1.4 The "Unique Names" (UN) rule

In a fictional story, one can have two characters with the same name, say "Paul", appearing in the same scene. This can make it confusing to follow the action of the story (which "Paul" is speaking?) but this is the author's choice.

In mathematical writing, this is not permitted. The unique names rule requires that once you introduce a letter, say x, to stand for an object, then during the life span of that object you may not use x for any other purpose except to refer to that object.

Earlier, we said that when x is introduced inside of a subproof, its life span ends at the conclusion of that subproof. This means that once the subproof is over, you are free to reuse the letter x, by introducing a x to represent some other object.

2 Dummy variables

The second major use of letters in writing mathematics is as *dummy variables* which are also called *temporary variables* or *bound variables.*

One way to define a dummy variable is that a dummy variable is a letter whose entire life span is one sentence or less.

Dummy variables are quite different from object names, but students get them confused all of the time. A dummy variable is a letter *temporarily* used to help define another letter or to make a statement. When a dummy variable is used, it typically varies over some set of values. When you use a dummy variable it is almost always possible to rephrase the sentence containing the dummy variable to avoid using it. The dummy variable is just a more convenient way to say what you want to say. Here are 3 examples:

1. Let $S = \{x \in \mathbf{Z} : x^2 + 1 \text{ is prime}\}$.

2. Let n be a natural number. Let s be $\sum_{k=1}^{n} k^2$.

3. For all $a, b \in \mathbf{N}$, $a + b$ belongs to $\mathbf{N}$.

In the first example, the sentence is an introduction for the set S, and S is an object name. On the other hand, x is a dummy variable. The variable x is only used to help explain what the set S is. Notice that it is possible to define S without the use of a dummy variable: "Let S be the set of those integers that have the property that the number obtained by adding 1 to its square is prime." Here we use the dummy variable x in order to define the set more easily.

In the second example, n and s are introduced as object names. k is a dummy variable that is used to help define s. The dummy variable k takes on all values in the set $\{1, \ldots, n\}$. We can describe s without the use of k by saying "Define s to be the sum of the squares of all consecutive integers starting from 1 and ending with n."

In the third example, a, b are dummy variables that are being used to help make a statement. Each variable takes on all values in $\mathbf{N}$. The statement can be made without reference to a and b by saying "The sum of any two natural numbers is a natural number".

> Let A be a finite set of positive integers. Then there exists an integer y that satisfies for all $z \in A$, $z|y$.

In the first sentence, A is an object name and in the second, y and z are dummy variables. The second sentence can be said without using y, z or w by saying "Then there is an integer that is a multiple of every member of A."

Another example:

> Let S be a subset of real numbers. Let T be the set of all real numbers t such that $2t \in S$.

Here S and T are object names and t is a dummy variable.

How do you know a variable is a dummy variable? There is no absolute rule, but it is almost always easy to tell. Here are some clear situations where the variable is a dummy variable.

- In any sentence of the form "$\forall x, A(x)$", or "$\exists x, A(x)$", x is a dummy variable.
- If we define a set using the notation $\{x : P(x)\}$ then x is a dummy variable.
- If j is an index of summation in a sum written using summation notation: $\sum_{j=1}^{w} f(j)$ where f is some function then j is a dummy variable. (But w is not a dummy variable!)
- The variable of integration in an integral is a dummy variable. Thus in the expression "Let $A = \int_{z=0}^{y} f(z)dz$, z is a dummy variable but A and y are not.

As stated earlier, if you can restate a sentence without letter, then the letter is a dummy variable.

The rules that must be followed with dummy variables are:

DVT rule Dummy variables are temporary rule.

DSON rule Don't steal object names rule.

UDN rules Unique Dummy variable name rule.

2.1 The "Dummy Variables are Temporary" (DVT) rule

The life span of a dummy variable is one sentence or less. When you use a letter as a dummy variable, this is very different than introducing it as an object name. After the sentence in which the dummy variable is used, you can not refer to the letter as an object.

For example, a common error is:

> Let $A = \{x \in \mathbf{R} : 0 < x < 1\}$. Then $x < 2$.

The variable x has not been explicitly introduced. Two correct ways to do this would be:

> Let $A = \{x \in \mathbf{R} : 0 < x < 1\}$. Then for all $x \in A$, $x < 2$.

or

> Let $A = \{x \in \mathbf{R} : 0 < x < 1\}$. Let $x \in A$. Then $x < 2$.

Notice that in the first correction, x is still a dummy variable. The second sentence of the first correction simply says that every element of A is less than 2. In the second correction, the second sentence introduces x as an object name for a specific element of x. Notice that in the second correction we can refer to x in later sentences since it has been introduced, but in the first correction we can't.

Here is another error that we saw earlier in the discussion of the EF rule.

> Let A be a finite nonempty set of natural numbers. Then there exists a natural number x such that $x \notin A$ and x is larger than any element of A. Let $y = x - 1$.

Here A is an object name. In the next sentence x is a dummy variable. In the next sentence we introduce the object name y, but we define it in terms of x. But x was never introduced.

As discussed earlier, the recommended way to correct this is:

> Let A be a finite nonempty set of natural numbers. Then there exists a natural number, which we will call x, such that $x \notin A$ and x is larger than any element of A. Let $y = x - 1$.

Here is a related error:

> For all even numbers n, there exists an integer which we will call k, such that $2k = n$.

Here the error is that k, which is being introduced as an object name is defined in terms of n which is a dummy variable. One way to see the error more clearly is to recall that using "which we will call ..." as in the above sentence is a shortcut. The above sentence has the same meaning as:

> For all even numbers n, there exists an integer j such that $2j = n$. Let k be an integer such that $2k = n$.

2.2 The "Don't Steal Object Names" (DSON) rule

This rule is simple. Once you are using x as an object name, you may not also use it as a dummy variable. For example:

> Let x and y be real numbers. For all $x, y \in \mathbf{R}$, $x + y = y + x$.

This is incorrect. In the first sentence you have introduced that x and y to be specific real numbers. In the second sentence, you use x and y as dummy variables. The following would be correct.

> Let x and y be real numbers. For all $a, b \in \mathbf{R}$, $a + b = b + a$. Therefore $x + y = y + x$.

or,

> Let x and y be real numbers. Then $x + y = y + x$, by the commutative law of addition.

2.3 The "Unique Dummy variable Name" (UDN) rule

In general you are allowed to reuse dummy variables. For example, you might write:

> Let $A = \{x \in \mathbf{R} : x^2 + x \geq 12\}$. Let $B = \{x \in \mathbf{R} : x^2 + x \geq 15\}$.

In this case, A and B are both object names while x is a dummy variable.
The following is also allowed:

> For all real numbers x that are greater than 1, $x^2 > x$ and for all real numbers x that are greater than 2, $x^2 > 2x$.

Here the same dummy variable is used in two different ways in the same sentence. This is something that should be done with great care. Whatever you write must make sense. Here are two examples that don't make sense, and are therefore not permitted.

> For all x, there exists x, such that $x^2 \geq x$.

> Let $A = \{n \in \mathbf{N} : \exists n \text{ such that } n \text{ is even}\}$.