Intro to Mathematical Reasoning (Math 300)

Supplement 6. Some sample problems (Version 10/8/03)

Here are some problems to give you some practice doing proofs. Full proofs for these are contained in Handout 7. You are strongly encouraged to try these on your own before reading the solutions.

Problem 6.1 Prove: For all integers a, b, c, if a is a divisor of b and b is a divisor of c then a is a divisor of c. (*Recall that x is a divisor of y means that there is an integer k such that $xk = y$.*)

Problem 6.2 Prove: No integer is both even and odd. (*Comment: Here you should use the following definition of odd: an integer n is odd provided that there is an integer j such that $n = 2j + 1$.*)

Problem 6.3 Each integer is even or odd.

Problem 6.4 Every rational number can be expressed as a ratio of two integers that have no common factor greater than 1.

Problem 6.5 There is no rational number whose square is equal to 2.

Problem 6.6 Let U be a set and let $\mathcal{A}$ be a set of subsets of U having the property that for all $A, B \in \mathcal{A}$, $A \cap B \neq \emptyset$. Then for all subsets Y of U, one of the following two conclusions hold: (1) For all $A \in \mathcal{A}$, $A \cap Y \neq \emptyset$, or (2) For all $A \in \mathcal{A}$, $A \cap Y^c \neq \emptyset$. (*Comment: Notice that in the statement of this theorem U, and $\mathcal{A}$ are introduced by "Let" statements. This means that the conclusion of the theorem should be true for any choice of U and $\mathcal{A}$ satisfying the hypotheses. Since U and $\mathcal{A}$ are introduced in the theorem statement, they are object names that can be referred to in the proof without further introduction. On the other hand, A, B and Y are dummy variables that can not be treated as object names.*)

Problem 6.7 Prove that for every positive real number R there is a real number z such that for all x belonging to the interval $(1 - z, 1)$, $1/(1 - x) \geq R$.