

# STOCHASTIC MODELS FOR OPERATIONS RESEARCH

## Math 424 (01:640:424)

**Time and Location:**

**Lecturer:**

Office:

Email:

Web Page:

Office Hours:

**Prerequisites:** Probability Theory (01:640:477 or 01:960:381)

**Textbook:** *An Introduction to Stochastic Modeling* (Third edition) by H. M. Taylor and S. Karlin. Academic Press, 1998. (ISBN: 0-12-684887-4)

**Course Webpage:** <http://www.math.rutgers.edu/courses/424/>

**Homework:** There will be six homework assignments. Late homework will not be accepted unless it is the result of an officially excused absence.

**Exams:** There will be one eighty-minute midterm exam and a three-hour cumulative final exam. Exams will be closed book and student-prepared formula sheets will not be permitted. An official formula sheet will be provided with each exam. The dates of the exams listed below are tentative. The actual dates will be announced in class. Make-up exams will be allowed provided that you can supply acceptable written evidence, and that you notify the lecturer *as soon as possible*.

**Midterm Exam:** To be announced.

**Final Exam:** To be announced.

**Grading:** The term grade will be based on the results of the examinations, and on the performance on the homework assignments. In summary, here are the components of the term grade:

Homework	30%
Midterm exam	30%
Final exam	40%

**Absences:** You are expected to attend every class. An absence due to illness or family emergency may be excused, provided that you can supply acceptable written evidence if required, and that you notify the lecturer *as soon as possible*.

**Course Outline:**

Chapters	Topics	Sections
I	Introduction–Probability Review	1.1-1.6
II	Conditional Probability and Conditional Expectation	2.1-2.5
III	Markov Chains: Introduction	3.1-3.6
IV	The Long Run Behavior of Markov Chains	4.1-4.5
V	Poisson Processes	5.1-5.6
VI	Continuous Time Markov Chains	6.1-6.6
VII	Renewal Processes	7.1-7.5
VIII	Brownian Motion and Related Processes; BSM Formula	8.1-8.4
IX	Queueing Systems	9.1-9.6

**General Information:**

**Markov chains:** definition, transition probabilities, special Markov chains (random walks, dams and inventories, branching processes), classification of states, limit theorems. **Poisson processes:** derivations, homogeneous, non-homogeneous processes, spacial and marked Poisson processes. **Continuous time Markov chains:** the Chapman-Kolmogorov equation, birth and death processes, the case of a finite state space, special cases, limiting behavior. **Renewal processes:** definition, the renewal function, replacement models, renewal theorems, inspection paradox, applications. **Brownian motions:** definition, processes with independent increments, the maximum variable and the reflection principle, Brownian bridge, geometric Brownian motion, applications in modern financial theory, Black-Scholes-Merton (BSM) formula for European call option. **Queueing theory:** queueing systems, Little's formula, Poisson arrivals and exponential and general service times, the case of an infinite number of servers, priority queues, queueing systems.

**Homework Assignments:** Additional homework problems will be assigned in class. Suggested homework problems are the following. Show your work to get full credit.

Homework	Suggested Problems
1	<i>Page 98</i> : Exercise 1.1 <i>Page 100</i> : Problems 1.3, 1.4 <i>Page 102</i> : Exercise 2.2 <i>Page 104</i> : Problems 2.2, 2.3 <i>Page 112</i> : Exercise 3.2 <i>Page 113</i> : Exercise 3.4 <i>Page 115</i> : Problem 3.9 <i>Page 127</i> : Exercise 4.2 <i>Page 150</i> : Problem 5.1 <i>Page 168</i> : Problem 6.2 <i>Page 183</i> : Exercise 8.1 <i>Page 184</i> : Problem 8.3
2	<i>Page 208</i> : Exercise 1.1 <i>Page 210</i> : Exercise 1.7 <i>Page 212</i> : Problem 1.5 <i>Page 243</i> : Exercises 3.1, 3.2 <i>Page 244</i> : Exercises 3.3, 3.4 <i>Page 245</i> : Problem 3.1(a)
3	<i>Page 274</i> : Exercise 1.6 <i>Page 275</i> : Exercise 1.7 <i>Page 276</i> : Problem 1.2 <i>Page 277</i> : Problem 1.9 <i>Page 294</i> : Exercise 3.3 <i>Page 308</i> : Exercise 4.1 <i>Page 328</i> : Exercises 6.1, 6.2 <i>Page 329</i> : Exercise 6.5
4	<i>Page 341</i> : Exercise 1.5 <i>Page 364</i> : Exercise 3.1 <i>Page 377</i> : Exercises 4.5, 4.6 <i>Page 377</i> : Problem 4.2
5	<i>Page 425</i> : Exercise 1.4 <i>Page 435</i> : Exercise 3.1 <i>Page 436</i> : Problems 3.3, 3.4 <i>Page 437</i> : Problem 3.5 <b>Hint:</b> Take $T = (-\infty, \infty)$ .
6	<i>Page 555</i> : Exercise 2.1 <i>Page 556</i> : Exercise 2.3 <i>Page 556</i> : Problems 2.2, 2.4