640:495 Mathematical Finance, Problems.

58. Show that if $X \sim N(m, \sigma^2)$, then $aX+b \sim N(am+b, a^2\sigma^2)$. (See, online notes on Normal random variables and the Central Limit Theorem, `http://www.math.rutgers.edu/courses/495/lect18notes.pdf`, page 2.)

59. Show that if $Y_1, \ldots, Y_n$ are independent and if each has finite variance, then $\mathrm{Var}\left(\sum_1^n Y_i\right) = \sum_1^n \mathrm{Var}(Y_i)$. (See, online notes on Normal random variables and the Central Limit Theorem, `http://www.math.rutgers.edu/courses/495/lect18notes.pdf`, page 2.)

60. If $Y \sim N(5, 16)$, find $\mathbb{P}(Y > 3)$.

61. For a loaded coin for which the probability of heads is 3/5. Find an approximate value for the probability that the number of heads in 100 flips is in the range $[55, 67]$.

62. Forty branches of a store report their sales daily to the central office. Assume that the error in the dollar amount reported by each store is uniformly distributed on the interval $[-100, 100]$ and that the errors from different stores is independent. What is the probability that the total error of the sum of the sales is greater than \$50?

63. Do problem 1, page 103 of the text. To do this problem you just need to use the Central Limit Theorem approximation. You can do this problem just from reading the class notes, but it will help you to read the discussion and examples on pp. 98-100.

64. Problem 2, page 103. Again this is a Central Limit approximation.

65. Do 1, 2, and 3 on page 70 of the text, each for (b) and (c) only. Assume in our notation that the last column of the table gives $e^{r\tau}$.

66. For the values of part (b) in the table on page 70 of the text, find the replicating portfolio process (the delta hedging values) for a look back option with payoff $\max\{S_0, S_1, S_2, S_3\}$.

67. Let $Z \sim N(0, 1)$. Calculate $E[e^{Z^2/8}]$.

68. Let $\{B_t\}_{t \geq 0}$ be a Brownian motion. Let $0 < s < t < u$. Compute $E[B_s B_t B_u]$. Hint: Condition first on $\{B_v, v \leq t\}$ and use independence and normality of increments.

69. Let B be a standard Brownian motion. Show that for any $s < t$, $E[B_t^2 - t \mid B_u, u \leq s] = B_s^2 - s$. This shows that $B_t^2 - t$ is a martingale.

70. Let $\{B_t\}_{t \geq 0}$ be a standard Brownian motion. Let $a > 0$. Show that $Y_y = (1/a) B_{a^2 t}$ is a standard Brownian motion.