The contents of Math 503

Each instantiation of Math 503 will likely be different, based primarily on the interests of the instructor and secondarily on the text chosen. The topics mentioned below will almost always be covered in any version of Math 503. Some suggestions are given for additional topics which may be included.

Note that 75% of the material is also commonly discussed in a U.S. *undergraduate* course. Although the topics may be the same, some of what distinguishes the treatment in Math 503 from what's done in such courses are the following considerations.

- Students are expected to give detailed proofs of the classical theorems. Most of the homework and exam problems will require proofs.
- Students are expected to be ardent apprentice analysts, and analysts should have mastery of *estimation*. So a major theme will be estimating functions, sums, integrals, etc. in order to verify geometric and analytic results.
- The central theorems of the subject (various Cauchy theorems, for example) can be stated for domains such as the disc and the plane, and for line integrals over circles and rectangles. While these results are sufficient for many applications, what is characteristic of introductory *graduate* complex analysis is statements of these results using suitable topological language (homotopy and/or homology) so that the results can be used in more advanced analytic situations and in applications to other areas.

List of topics

- Definition of holomorphic function using (in some order!) power series and Cauchy-Riemann equations and existence of a complex derivative.
- The Cauchy integral formula; Cauchy's Theorem; Morera's Theorem.
- Analysis of isolated singularities: removable, pole, essential.
- The Residue Theorem.
- Standard applications, including the Cauchy inequalities, the Maximum Modulus Principle, evaluation of real integrals and sums of series, zero sets of holomorphic functions, Rouché's Theorem and the Argument Principle, $\star$analytic continuation$\star$, and $\star$the Phragmén-Lindelöf Theorem$\star$.
- Limits of holomorphic functions.
- Elementary theory of harmonic functions.
- Conformal mapping, including the automorphisms of $\mathbb{C}$ and the Riemann sphere and the unit disc (linear fractional transformations).
- Statement and $\star$proof$\star$ of the Riemann Mapping Theorem.
- $\star$Prescribing the zeros and poles of an analytic function (some versions of the Mittag-Leffler and Weierstrass Theorems)$\star$.
- Understanding the analytic and geometric properties (over $\mathbb{C}$) of polynomials, rational functions, the exponential and logarithmic functions, and $\star$other transcendental functions$\star$.
- $\star$Elementary (intuitive) Riemann surfaces$\star$.

Topics parenthesized with $\star$'s may not be covered precisely the same way each semester.