Suggested Background for Math 551

Many of the topics in Math 551 are covered in the advanced undergraduate courses Math 451 and 452, but in Math 551, they are covered more deeply and abstractly (and in one semester rather than two), and students taking Math 551 are assumed to already be reasonably familiar with the basics of these topics. Familiarity with examples is particularly important.

In Math 451 and 452, Artin's "Algebra" (selected parts) was used in 2006-07, but any equivalent book can certainly be used. Below are the topics (from Artin's book) that were covered (sometimes in ways a little different from the ways they are treated in the text) in 2006-07:

- Chapters 1, 3 and 4 deal with material that we teach in courses (Math 250, Math 350) on linear algebra and so may be covered rather quickly. However, this material is quite important for the development of examples (e.g., groups and rings of matrices and linear transformations).

- Chapters 2 and 6 - except section 6.9 - (on group theory) are covered in detail. As much of Chapter 5 (symmetry) as time permits is covered and Sections 8.1 - 8.4 (linear groups) are also covered as time permits.

- Chapter 10 (Rings), Sections 1-7: Definition of ring, formal construction of polynomial rings, homomorphisms and ideals, quotient rings and relations in a ring, adjunction of elements, integral domains and fields of fractions, maximal ideals

- Chapter 11 (Factorization), Sections 1-4: Factorization of integers and polynomials, unique factorization domains and principal ideal domains, Gauss's lemma and consequences, explicit factorization of polynomials

- Chapter 12 (Modules), Sections 1, 2 and 4-7: Definition of module, matrices, free modules and bases, diagonalization of integer matrices, generators and relations for modules, the structure theorem for finitely generated abelian groups, application to linear operators: Jordan canoncal form

- Although Chapter 13 (Fields) was not covered in 2006-07, it would be a good idea to be somewhat familiar with the beginning of this chapter: Examples of fields, algebraic and transcendental extensions, the degree of a field extension

It would of course be a good idea to do as many exercises as possible, but without feedback on solutions, this could be difficult. Artin's text does omit some proofs, leaving them as exercises, and it would be a very good idea to fill in such proofs.