

Oral Qualifying Exam Syllabus

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1 Combinatorics

- **Enumeration:** Counting Arguments, Generating Functions, Rook Polynomials, Inclusion Exclusion
- **Set Systems:** Sperner's Theorem, LYM Inequality, Erdős-Ko-Rado, Kruskal-Katona, Fisher's Inequality, Raychaudhuri-Wilson, Frankl Wilson, Baranyai's Theorem
- **Lattices and Posets:** Dilworth's Theorem, Birkhoff's representation theorem, Möbius Inversion, Weisner
- **Ramsey Theory:** Ramsey's Theorem, infinite Ramsey theory, probabilistic lower bounds, van der Waerden
- **Discrepancy Theory:** Beck-Fiala, Six Standard Deviations Suffice
- **Algebraic Methods:** Dimension Arguments, Graham-Pollak, Combinatorial Nullstellensatz
- **Experimental Mathematics:** Maple Programming

2 Graph Theory

- **Matching:** Hall's Theorem, König's Theorem
- **Connectivity:** Menger's Theorem, Max-Flow Min-Cut, Kruskal's Algorithm
- **Coloring:** Brook's Theorem, Vizing's Theorem, 5-Color Theorem, Hadwiger's Conjecture
- **Extremal Problems:** Turan's Theorem, Statement of The Regularity Lemma, Erdős Stone
- **Random Graphs:** Number of triangles, clique number, threshold for connectedness
- **Planar Graphs:** Euler Characteristic, Proof that K_5 and $K_{3,3}$ are not planar, Statement of Kuratowski's Theorem

3 The Probabilistic Method

- **Basics:** Linearity of Expectation, Markov, Chebyshev, Chernoff bound, Law of Total Probability
- **Alterations:** Property B, Graphs with high girth and chromatic number.

- **Second Moment Method:** Threshold Functions for subgraph problem, Number of Prime Factors.
- **Lovasz Local Lemma:** Symmetric and general versions, Ramsey bounds, Property B, hypergraph discrepancy

4 Number Theory

- **Dirichlet Series:** Definition of Dirichlet Series, Zeta Function, and L function, Dirichlet's Theorem on Primes in Arithmetic Progressions
- **Modular Forms:** Definition of modular group, modular form, modular function, and Hecke Operator. Properties of the Vector Space of Modular Forms, Eigenfunctions of Hecke Operators. Theta functions and representations by quadratic forms.