Oral qualifying exam syllabus
Bence Borda

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1. Combinatorics

Enumeration: double counting, pigeonhole principle, recurrence relations, generating functions, inclusion-exclusion, Stirling numbers
Hypergraphs: Sperner, LYM-inequality, Erdős–Ko–Rado, Kruskal–Katona, Fisher, Ray–Chaudhuri–Wilson, Baranyai, Beck–Fiala, projective geometries
Posets and lattices: Dilworth, graded, modular and distributive lattices, Birkhoff representation theorem, Möbius inversion
Ramsey-theory: Ramsey, Chvátal–Rödl–Szemerédi–Trotter, Van der Waerden and Szemerédi on arithmetic progressions
Infinite combinatorics: Kőnig-lemma, compactness
Inequalities: Harris, FKG, Ahlswede–Daykin, entropy, Shearer's lemma
Algebraic methods: linear algebra methods, combinatorial nullstellensatz

2. Graph theory

Matchings: Kőnig, Hall, Tutte, algorithm for maximal matching
Colorings: Brooks, Vizing, 5 color theorem
Extremal graph theory: Turán, Erdős–Stone–Simonovits
Flow networks: max-flow min-cut, Ford–Fulkerson algorithm, Menger, applications
Algebraic methods: adjacency and incidence matrix, Cayley
Regularity lemma: statement of the regularity lemma, triangle removal lemma

3. The probabilistic method

Methods: union bound, Bonferroni inequalities, linearity of expectation, Markov and Chebychev inequalities, Chernoff bound, alterations, Lovász local lemma, Janson inequality

Random graphs: monotone properties, existence of threshold functions, lower bound on Ramsey numbers, number of triangles in $G_{n,p}$, threshold function for containing a fixed subgraph, graphs with high chromatic number and high girth

4. Probability theory

Probability spaces: σ-algebras, independence, Kolmogorov zero-one law, Borel–Cantelli lemma
Random variables: independence, distribution, distribution function, density function, characteristic function, expected value and conditional expected value, variance, median
Inequalities: Markov, Chebychev, Chernoff, Lévy
Convergence of random variables: stochastic, with probability 1, in L_p, in distribution, uniform integrability
Laws of large numbers: weak and strong laws of large numbers, Feller, Kolmogorov
Central limit theorem: weak convergence of probability measures, continuity theorem of characteristic functions, central limit theorem, Lindeberg's condition
Martingales: discrete time submartingales and supermartingales, stopping times, convergence of submartingales, upcrossing inequality, maximal inequality and Kolmogorov's inequality for submartingales