Oral Qualifying Exam Syllabus – Final Draft – October 7, 2013

Pat Devlin

Fall 2013

1 Primary Topic: Combinatorics

1.1 Enumerative combinatorics

Basic enumeration: counting arguments, pigeon-hole principle, inclusion-exclusion, recurrence relations, Stirling's formula, binomial coefficients, Stirling numbers, Bell numbers, Fibonacci numbers, Catalan numbers

generatingfunctionology: ring of formal power series, complex Laurent series, ordinary generating functions, exponential generating functions, rational generating functions, quasi-polynomials, Lagrange-inversion formula, sequence asymptotics

Set systems: Sperner's theorem, Erdős-Ko-Rado, Kruskal-Katona, Fisher's and generalized Fisher's inequalities, uniform and nonuniform Ray-Chaudhuri–Wilson, Frankl–Wilson, LYM inequality, Harper's theorem, Baranyai's theorem

Lattices and posets: Dilworth, distributive lattices, Birkhoff representation theorem, Dilworth's theorem on dimension of distributive lattices, geometric lattices, Möbius inversion, Weisner, Dowling-Wilson, linear extensions of posets

Correlation inequalities: Harris-Kleitman, Fortuin-Kasteleyn-Ginibre, Ahlswede-Daykin, XYZ theorem

Algebraic methods: inclusion matrices and application to Gasparian's proof of the weak perfect graph theorem, Combinatorial Nullstellensatz, Chevalley-Warning and application to Alon-Dubiner, Erdős-Ginsberg-Ziv

Ramsey theorey: Ramsey's theorem, infinite Ramsey, König's tree lemma, probabilistic lower bounds, van der Waerden, Hales-Jewett, statement of Szemerédi's theorem

Discrepancy: Beck-Fiala, Roth's $\frac{1}{4}$-theorem on arithmetic progressions, 'six standard deviations'

Linear Programming: Weak duality theorem, strong duality theorem, complementary slackness, examples, fractional coverings and matchings

1.2 Graph theory

Matching: König, Hall, Tutte's 1-factor theorem, matching polytope, Gale-Shapley algorithm for stable matchings

Connectivity: Kruskal's alorithm for minimum weight spanning tree, structure of 2-connected graphs, Menger, max-flow-min-cut

Planarity: Euler's formula, Kuratowski's theorem, Wagner's theorem

Coloring: 5 color theorem, Brooks, Vizing, Thomassen's 5-list-coloring of planar graphs, Galvin's proof of Dinitz conjecture, perfect graphs, Lovász's proof of weak perfect graph conjecture

Extremal problems: Turán, statement of regularity lemma, Erdős-Stone, Chvátal-Rödl-Szemerédi-Trotter

1.3 Probabilistic method

Basics: linearity of expectation, Bonferroni inequalities, common distributions, conditional probability, law of total probability, Chernoff bounds, Chebyshev inequality

Alteration method: general procedure, basic examples (independent sets, packing, triangles in the unit square), application to Property B (hypergraph coloring)

Second moment method: general procedure, application to threshold functions

Lovász local lemma: symmetric and general versions, Ramsey lower bounds, hypergraph coloring, application to Latin transversals

Poisson paradigm: Janson inequalities and applications to the number of triangles in $G_{n,p}$, Brun's sieve and application to the number of isolated vertices in $G_{n,p}$

Martingales: Definitions, Azuma's inequality, application to chromatic number, Talagrand's inequality, comparing Talangrand's inequality and Azuma's inequalities, longest increasing subsequence problem, vertex edge exposures, application to $\chi(G)$

Random graphs: $G_{n,p}$ versus $G_{n,M}$, monotone properties, existence of theshold functions, Bollobás-Thomason, probabilistic refutation of the Hájos conjecture

Entropy: basic properties, Shearer's Lemma, applications to Minc Conjecture

1.4 Experimental and computational methods

Basic concepts: Maple, C++, general algorithms, running times, data structures, memoization

Applications to sequences: computing and solving recurrences, conjecturing asymptotics, C-finite ansatz (constant coefficients), holonomic ansatz (polynomial coefficients)

Applications to combinatorial games: computing P/N-positions, finding Sprague-Grundy numbers, games on graphs, sums of games

2 Secondary Topic: Probability

Probability spaces and random variables: algebras and σ-algebras, probability spaces, monotone class theorem, independence and product spaces, random variables, distribution functions, expectation, independence of random variables, convergence concepts for random variables, Kolmogorov's Zero-One Law

Large number laws: weak law of large numbers, Borel-Cantelli Lemma, strong law of large numbers, Kolmogorov's three series theorem

Central limit theorem: De Moivre-Laplace, weak convergence and convergence in distribution, characteristic functions, continuity theorem, Lindeberg-Feller

Martingales: conditioning, conditional expectation, sub/super-martingales, stopping times, optional stopping theorems, application to random walks, Doob's martingale inequalities, Doob's upcrossing inequality, uniform integrability, martingale convergence theorems, Levy's upward convergence theorem