Syllabus for Oral Qualifying Exam

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I. Mean Curvature Flow

1. Definition of the Mean Curvature Flow:
- First Variation of the Area Funtional
- Special Solutions
- Short Time Existence of the Flow

2. Evolution of Geometric Quantities:
- Maximum Principle
- Comparison Principle
- Evolution of Curvature
- Consequences of Evolution Equations
- Convexity Invariance

3. Monotonicity Formula and Type I Singularities:
- The Monotonicity Formula and Integral Estimates
- Type I Singularities and the Rescaling Procedure
- Analysis of Singularities
- Hypersurfaces with Nonnegative Mean Curvature

4. Type II Singulaties:
- Hamilton's Blow-up
- Hypersurfaces with Nonnegative Mean Curvature
- Hamilton's Harnack Estimates for Mean Curvature Flow

4. Regularity Theory at the First Sinfular Time:
- Lower Bound on Area Ratio / Clearing Out Lemma

- White's Gap Theorem
- Brakke's Regularity Theorem under Area Continuity and Unit Density Hypothesis

II. **Partial Differential Equations**

1. Sobolev Spaces
- Holder and Sobolev spaces
- Approximation
- Extensions
- Traces
- Sobolev inequalities
- Compactness

2. Laplace's Equation
- Fundamental Solution
- Mean-Value Formulas
- Properties of Harmonic Funtions
- Green's Function
- Energy Methods

3. Second-Order Elliptic Equations
- Existence of Weak Solutions
- Regularity
- Maximum Principles
- Eigenvalues and Eigenfuncitons

4. Heat Equation
- Fundamental Solution
- Mean-Value Formula
- Properties of Solutions
- Energy Methods

5. Second-Order Parabolic Equations
- Existence of Weak Solutions
- Regularity
- Maximum Principles

References

[Ec] Klaus Ecker, Regularity Theory for Mean Curvature Flow

[Ev] Lawrence C. Evans, Partial Differential Equations

[M] Carlo Mantegazza, Lecture Notes on Mean Curvature Flow.