

EXAM SYLLABUS

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1. ALGEBRAIC NUMBER THEORY

- (1) Number Fields: Integral Bases, Different, Discriminant
- (2) Ramification, Splitting of Primes
- (3) Ideal Class Group, Minkowski Bound, Finiteness of the Class Number
- (4) Dirichlet's Unit Theorem
- (5) Quadratic and Cyclotomic Fields

2. ALGEBRAIC GEOMETRY

- (1) Sheaves, Schemes, Sheaves of Modules
- (2) Divisors, Riemann-Roch for Curves
- (3) Differentials, Genus
- (4) Cohomology of Sheaves, Cohomology of Noetherian Affine Schemes and projective space, Serre Duality

3. LINEAR ALGEBRAIC GROUPS

- (1) Correspondence Between Groups and Lie Algebras in Characteristic Zero
- (2) Jordan-Chevalley Decomposition, Diagonalizable Groups
- (3) Reductive and Semisimple Groups
- (4) Structure of Borel Subgroups
- (5) Weyl Group
- (6) Bruhat Decomposition
- (7) Parabolic Subgroups
- (8) Examples of Classical Groups
- (9) Root Systems and Dynkin Diagrams

4. ELLIPTIC CURVES

- (1) Elliptic Curves over \mathbb{C}
- (2) Elliptic Curves over finite fields, The Hasse Bound
- (3) Elliptic Curves over local fields
- (4) Elliptic Curves over Global fields, The Mordell-Weil Theorem