Oral Qualifying Exam Syllabus

John Kim

1 Combinatorics

Basic Enumeration Counting Arguments, Binomial Coefficients, Recurrence Relations, Principle of Inclusion-Exclusion, Stirling's Formula, Generating Functions

Set Systems Sperner's Lemma, LYM Inequality, Erdos-Ko-Rado, Kruskal-Katona, Fisher's Inequality, Frankl-Wilson

Lattices and Posets Dilworth's Thm, Distributive and Geometric Lattices, Birkhoff Representation Thm, Mobius Inversion, Weisner, Dowling-Wilson

Correlation Inequalities Harris-Kleitman, FKG Inequality, Ahlswede-Daykin Four Function Thm,

Ramsey Theory Ramsey's Thm, Infinite Ramsey Theory, Probabilistic Lower Bounds, Van der Waerden

Discrepancy Beck-Fiala

2 Graph Theory

Matching Konig's Thm, Hall's Thm, Tutte's 1-factor Thm

Connectivity Kruskal's Algorithm for Min Weight Spanning Tree, Menger's Thm, Max-Flow Min-Cut

Planarity Euler's Formula, Crossing Number, Wagner's Thm, Kuratowski's Thm

Coloring Brook's Thm, Vizing's Thm, 5-Color Thm

Extremal Problems Turan's Thm, Erdos-Stone Thm, Szemeredi's Regularity Lemma, Quasirandom Graphs

3 Probabilistic Method

Basics Bonferroni Inequalities, Markov and Chebyshev Inequalities, Chernoff Bounds, Method of Alterations, Graphs with High Girth and Chromatic Number

Second Moment Method Threshold functions in Random Graphs, Distinct Sums

Lovasz Local Lemma Symmetric and General Versions, Ramsey Number Lower Bounds

Martingales Vertex and Edge Exposure, Azuma's Inequality, Chromatic Number

Poisson Paradigm Janson Inequalities, Number of Triangles in a Random Graph, Number of Isolated Points

4   Probability Theory

Probability Spaces and Random Variables   Algebras and σ-Algebras, Probability Spaces, Monotone Class Thm, Independence, Product Spaces, Random Variables, Distribution Functions, Expectation, Types of Convergence, Kolmogorov's $0-1$ Law

Laws of Large Numbers   Weak Law of Large Numbers, Borel-Cantelli Lemma, Strong Law of Large Numbers, Kolmogorov's 3-Series Thm

Stationary Processes   Measure-Preserving Transformations, Birkhoff's Ergodic Thm, Ergodic Thm for Stationary Processes

Central Limit Theorem   De Moivre-Laplace Thm, Weak Convergence and Convergence in Distribution, Continuity Thm, Lindeberg-Feller

Martingales   Conditional Expectation, Sub/Super-Martingales, Stopping Times, Optional Stopping Thm, Random Walks, Doob's Martingale Inequalities, Doob's Upcrossing Inequality, Uniform Integrability, Martingale Convergence Thms