Syllabus for Oral Examination
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Stochastic Control

1. Dynamic programming and HJB equations

   - Bellman's principle of optimality
   - The HJB equation
   - Properties of the value function (continuous dependence on parameters, semiconcavity)
   - Viscosity solutions

2. Verification theorems

   - Smooth case
   - Nonsmooth case
   - Merton's portfolio problem

3. Filtering theory

   - The Kalman-Bucy filter
   - Nonlinear filtering for SDEs (the Kallianpur-Striebel formula, the Zakai equation)

4. Linear quadratic optimal control problems

   - Solutions for finite time and time-average cost
   - Stochastic Riccati equations
   - Existence

Viscosity Solutions

1. The notion of viscosity solutions

2. The maximum principle for semicontinuous functions and comparison for the Dirichlet problem

3. Perron's method and existence

4. Comparison

   - Comparison with more regularity
   - Estimates from comparison
   - Comparison with strict inequalities and without coercivity in u
   - Comparison and existence of unbounded solutions on unbounded domains

5. Limits of viscosity solutions

6. General and generalized boundary conditions

   - Boundary conditions in the viscosity sense
   - Existence and uniqueness for the Neumann problem
   - The generalized Dirichlet problem
   - Fully nonlinear boundary conditions

References

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