Oral Qualifying Exam Syllabus
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Functional Analysis and Operator Theory

1. Banach Spaces
    a. Continuity of Bounded Linear Operators as a Function on Weak Topologies
    b. Sequential Compactness of Reflexive Spaces
    c. Banach-Alaoglu Theorem
    d. Metrizability of of the Unit Ball in the Weak Topology When the Dual Space is Separable
    e. Reisz's Lemma
    f. Compactness of The Unit Ball in the Strong Topology
    g. Hahn-Banach Theorem
2. Hilbert Spaces
    a. Projection Lemma
    b. Riesz Representation Theorem
    c. Stampacchia and Lax-Milgram
    d. Hilbert Sums and Orthonormal Bases
3. Compact Operators on a Hilbert Space
    a. Compact Operators as the Closure of Finite Rank Operators
    b. Fredholm Alternative
    c. Riesz-Schauder Theorem
    d. Spectral Decomposition of Self-Adjoint Compact Operators

Partial Differential Equations and Concentration Compactness

1. Laplace Equation
    a. Fundamental Solution
    b. Properties of Harmonic Functions: Mean Value Property, Maximum Principles, Regularity, Liouville's Theorem, Harnack's Inequality
    c. Green's Function on the Half-Space and the Ball
    d. Energy Methods: Uniqueness and Dirichlet's Principle
2. Existence of Weak Solution of Second Order Elliptic Equations
    a. Lax Milgram Theorem
    b. Energy Methods
    c. Fredholm Alternative
3. Concentration Compactness Theorem
    a. Sobolev Inequality
    b. Application to Sobolev Inequality to Prove Existence of Minimizers
    c. Helly Selection Principle
    d. Rellich Theorem
    e. Lebesgue-Radon-Nikodym Theorem
    f. Brezis-Lieb Lemma