Qualifying Exam Syllabus

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1 Combinatorics

Basics: Bijective proofs, the translation method, involution principle, binomial coefficients and their identities, inclusion-exclusion, Stirling's formula, Stirling numbers of first and second kind, Catalan numbers.
Generating functions: ordinary generating functions, exponential generating functions, exponential formula, Dirichlet series, Lagrange inversion, Polya's counting theorem.
Recurrences: linear recurrences (C-finite, P-finite), systems of first-order linear recurrences, solution of linear recurrences
Posets: chains and antichains, graded posets, lattices, distributive lattices, geometric lattices, Birkhoff representation theorem, Dilworth's theorem, incidence algebras.
Ramsey theory: Ramsey's theorem for graphs and hypergraphs, upper and lower bounds, van der Waerden's theorem
Hypergeometric functions: definitions, Sister Celine's method, Gosper's Algorithm, WZ algorithm.

References

Stanley, *Enumerative Combinatorics*, vols. 1, 2
Graham, Knuth, and Patashnik, *Concrete Mathematics*
Wilf, *Generatingfunctionology*
Ptekovsek, Wilf, and Zeilberger, $A = B$

2 Probabilistic method

Probability theory: Kolmogorov axioms, expected value, variance, moments, binomial distribution, Poisson distribution, central limit theorem, Chernoff bound

Basics: Basic probabilistic method, linearity of expectation, law of total probability, method of alterations, Markov's inequality
Second-moment techniques: Second-moment method, Markov's inequality, Chebyshev's inequality
Lovász local lemma: Symmetric version, asymmetric version, effective version of Moser and Tardos
Martingales: Definition, Azuma's inequality, vertex and edge exposure.
Quasirandom graphs: Definition, equivalence of conditions for quasirandomness.

References

Alon and Spencer, *The Probabilistic Method*
Moser and Tardos, "A constructive proof of the general Lovász local lemma," *JACM* 57 (2010)

3 Graph theory

Matchings: König's theorem, Hall's theorem, Tutte's theorem, theorem of Gallai and Milgram
Connectivity: Menger's theorem, max-flow min-cut theorem, Prim's algorithm, Kruskal's algorithm, matrix-tree theorem
Planar graphs: Euler's formula, Kuratowski's theorem, Wagner's theorem
Coloring: Brooks' theorem, Vizing's theorem, 5-color theorem
Extremal graph theory: Turan's theorem, statement of regularity lemma and application to Erdős-Stone theorem

References

Bollobás, *Modern Graph Theory*
Diestel, *Graph theory*

4 Combinatorial game theory

Fundamentals: Surreal numbers, games
Impartial games: Nimbers, Sprague-Grundy theorem, misere play of impartial games, impartial games wtih loops
Partisan games: Infinitesimal and infinite surreal numbers and games, disjunctive sums, thermography

Computational methods: A* search, heuristic functions, adversarial search, alpha-beta pruning, evaluation functions

References

Conway, *On Numbers and Games*
Berlekamp, Conway, and Guy, *Winning Ways for Your Mathematical Plays*
Russell and Norvig, *Artificial Intelligence: A Modern Approach*