Syllabus for Oral Examination

Liming Sun

1 Major Topic: Partial Differential Equation

• Classical solution of laplace equation

(1) Fundamental solution
(2) Mean vaule property
(3) Maximum principle
(4) Green's representation formula
(5) Harnack's inequality
(6) Louville theorem
(7) Perron's method
(8) Hopf lemma
(9) Schauder estimate
(10) Existence results by the method of continuity and fredholm alternative

• Classical solution of heat equation

(1) Fundamental Solution
(2) Maximum Principles on bounded $\overline{\Omega}_T$ and $R^n \times [0, T]$
(3) Backward uniquness by energy methods.

• Wave equation

(1) d' Alembert's formula, Kirchoff's formula.
(2) Uniqueness and domain of dependence by energy methods.

• Sobolev space

(1) Definition
(2) Density
(3) Extensions and Traces
(4) Rellich-Kondrachov compact imbedding theorem
(5) Poincare's inequality and Morrey's inequality
(6) $W_0^{1,n}$ case

- **Weak solution of second order Elliptic equation**
(1) Lax-Milgram theorem and Fredholm alternative
(2) Maximum principle
(3) Moser Interation method and Hanack's inequality
(4) L^p estimate and Interpolation theorem
- **Weak solution of second order Parabolic equation**
(1) Defintion of weak solution
(2) Energy estimates, Galerkin approximation
(3) Existence and uniqueness.
(4) Maximum principle.

2 Minor Topic: Riemannian Geometry

- Riemannian metric

- Levi-civita connection

- Curvature tensor, sectional curvature, ricci curvature, scalar curvature

- Geodesics and exponential map

- Jacobi fields, conjugate points

- First variation and second variation of arc length

- Bonnet-Myers Thm, Hopf-Rinow Thm, Cartan-Hadamard Thm

- Space form

- Hypersurface, second fundamental form

- Differential operator: grad, div and Hess.

References

[1] Gilbarg, D and Trudinger, N. S, Elliptic Partial Differential Equations of Second Order. Springer 1983

[2] Evans, L.C. Partial Differential equations. AMS Providence, 1998.

[3] Shoshichi Kobayashi, Katsumi Nomizu. Foundations of Differential Geometry. Interscience Publishers, 1963.