Oral Examination Program

Semeon Artamonov

Committee: Prof. Y.-Z. Huang, Prof. J. Lepowsky, Prof. V. Retakh (Chair), Prof. S. Sahi.

1 Major Topic: Noncommutative Algebras and Representation Theory

1. Quasideterminants

    (a) Definition and basic properties.

    (b) Noncommutative Plücker coordinates.

    (c) Factorization of Vandermonde Quasideterminants.

    (d) Noncommutative symmetric functions.

    (e) Noncommutative traces and cyclic vectors.

2. Infinite-dimensional Lie algebras

    (a) Affine Lie Algebras:

        i. Definition and basic examples. Affine Dynkin diagrams, classification.
        ii. Coxeter groups. Affine Weyl groups.
        iii. Realization of affine Lie algebras.

    (b) Extended Affine Lie Algebras

        i. Definition and basic examples, Multiloop Algebras.
        ii. Central and Cocentral extensions.
        iii. Reflection Systems, Semilattices.
        iv. Extended Affine Root Systems.
        v. Extended Affine Weyl Groups.

3. Noncommutative Poisson Geometry

    (a) Homology of Lie Algebras.

    (b) Left and Right Loday (Leibnitz) brackets.

    (c) Van den Bergh approach for construction of examples Loday brackets. Yang-Baxter equation for double brackets.

    (d) Poisson brackets on the moduli space of representations from double brackets.

4. Integrable systems

    (a) Liouville Integrable Systems. Definition and Examples.

    (b) Lax equation, examples.

    (c) Noncommutative Toda chain.

2 Minor Topic: Vertex Operator Algebras

1. Definitions and Properties

    (a) Formal calculus
    (b) Definition of Vertex Operator Algebra and basic properties
    (c) Weak Commutativity and Associativity of Vertex Operator Algebras
    (d) Definition of Modules and their basic properties.

2. Examples of Vertex Operator Algebras

    (a) VOA and modules associated with Virasoro algebras.
    (b) VOA and modules associated with affine Lie algebras.
    (c) VOA and modules associated with Heisenberg Lie algebras.

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