

# ORAL QUAL SYLLABUS

## 1. LOGIC

### 1.1. Model Theory.

- Basics
  - Compactness, Lowenheim-Skolem theorems, Tarski-Vaught Test
- Fraïssé
  - General theory
  - Quantifier elimination
  - Existence proofs
  - Classification (homogeneous tournaments)
  - Homogeneous  $k$ -dimensional permutations
- $\aleph_0$ -Categoricity
  - Omitting types
  - Atomic, prime, homogeneous, and saturated models
  - Oligomorphic automorphism group
- Order Indiscernibles
  - Existence
  - Locally finite quadrangles
  - Stability implies true indiscernibles
- Morley Rank
  - Definition in terms of types (Cantor-Bendixson)
  - Definition in terms of definable sets
  - $\aleph_1$ -categoricity  $\Rightarrow$   $\aleph_0$ -stability  $\Leftrightarrow$  Morley rank is defined
  - Rank 1, degree 1  $\Leftrightarrow$  strongly minimal
  - Strongly minimal geometry
  - Strongly minimal  $\Rightarrow$   $\aleph_1$ -categorical

### 1.2. Descriptive Set Theory.

- Polish Spaces
  - Borel isomorphism theorem
  - Borel-generated topologies, Ramsey-Mackey theorem
  - Sequential trees
- Borel and Projective Hierarchies
  - Basic definitions and facts, including closure properties
  - Existence of universal sets, non-collapsing
  - Every Polish space contains an analytic set that is not Borel
  - Equivalence of various definitions of analytic sets
  - Regularity properties of analytic sets

### 1.3. Forcing.

- Statements of fundamental forcing theorems
- Force CH, force  $\neg$ CH, force  $\diamond$
- Chain and closure conditions
- Cohen forcing
- Martin's axiom
- Product forcing
- Easton's theorem

## 2. COMBINATORICS

- **Enumeration:** bijections, binomial and multinomial coefficients, generating functions, recurrence relations, inclusion-exclusion
- **Extremal Results:** Sperner's theorem, Dilworth's theorem, Erdos-Ko-Rado
- **Probabilistic Method:** linearity of expectation, union bound, Chebyshev's inequality, Chernoff bounds, Lovasz local lemma
- **Ramsey Theory:** Ramsey, infinite Ramsey, probabilistic lower bounds, statement of van der Waerden
- **Linear Programming:** duality, combinatorial min-max theorems
- **Entropy:** basic properties, Shearer's lemma, Bregman's theorem
- **Algebraic Methods:** Schwartz-Zippel Lemma, Combinatorial Nullstellensatz