Oral Qualifying Exam Syllabus

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1 Functional Analysis

1. Banach spaces
   - Theorems of Hahn-Banach, open mapping, closed graph, Baire category, Banach-Alaoglu, and the uniform boundedness principle
   - Weak and weak* topologies
   - Bounded, unbounded, closed, and compact operators
   - Reflexive and separable spaces
2. Hilbert spaces
   - Orthogonality, projections, Fourier series, Riesz representation
3. Spectral theory
   - Spectrum, resolvent, and the Fredholm alternative for compact operators
   - Functional calculus and the spectral theorem for compact, bounded, and unbounded self-adjoint operators
   - Types of spectra; pure point, absolutely continuous, continuous singular
   - Characterization of self-adjoint operators
     - Stone's theorem
     - Kato-Rellich theorem

2 Partial Differential Equations

1. Fourier transform
2. Laplace equation
   - Mean value property, maximum principle, fundamental solution, Green's functions, Perron's method, and energy methods
3. The spaces $H^1(\mathbb{R}^n)$ and $H^{1/2}(\mathbb{R}^n)$; weak derivatives, approximation by smooth functions

3 References

1. Evans, Lawrence. *Partial Differential Equations.*
2. Lieb, Elliot H. and Loss, Michael. *Analysis*, 2nd ed.
3. Reed, Michael and Simon, Barry. *Methods of Mathematical Physics I: Functional Analysis.*