Oral Qualifying Exam Syllabus
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1 Partial Differential Equations

(a) Laplace/Poisson's Equation

    (i) Fundamental Solution

    (ii) Properties of Harmonic Functions

    (iii) Harnack Inequality

    (iv) Green's Functions

    (v) Energy Methods

    (vi) Regularity

(b) Second Order Elliptic Equations

    (i) Existence of Weak Solutions (Lax-Milgram)

    (ii) Regularity

    (iii) Maximum Principles (non-divergence equations)

    (iv) Eigenvalues/vectors.

(c) Heat Equation

    (i) Fundamental Solution

    (ii) Duhamel's Principle

    (iii) Mean value formula

    (iv) Regularity

    (v) Backwards Uniqueness

(d) Wave Equation

    (i) d'Alembert, Kirchoff

    (ii) Method of Spherical Means

    (iii) Uniqueness and Domain of Dependence.

(e) Sobolev Spaces

    (i) Definition and density of smooth functions.

    (ii) Extensions and Traces

(iii) Rellich-Kondrachov Embedding

(iv) Poincare's and Morrey's inequality, Gagliardo-Nirenberg-Sobolev.

(f) Other Topics

(i) The space H^{-1}.

(ii) Difference Quotients.

(iii) Fourier Methods.

References:

- Lawrence C. Evans, *Partial Differential Equations*, Second Edition. Chapters 2, 5, 6.

- MATH 517 and MATH 518 Class Notes.

2 Functional Analysis

(a) Banach and Hilbert Spaces

(i) Hahn-Banach Theorem

(ii) Uniform Boundedness Principle

(iii) Open Mapping Theorem and Closed Graph Theorem

(iv) Dual Spaces and Reflexive Spaces

(v) Weak and Weak* topology.

(vi) Projection Lemma, Riesz Representation.

(vii) Baire Category Theorem and applications.

(viii) Banach-Alaoglu Theorem

(b) Bounded Operators and the Spectral Theorem

(i) Adjoints of operators.

(ii) Spectrum and properties of it.

(iii) Compact operators.

(iv) Fredholm Alternative.

(v) Spectral Theorem.

References:

- Haim Brezis, *Functional Analysis, Sobolev Spaces, and Partial Differential Equations*. Chapters 1-6, 8-9.

- MATH 507 Class Notes.