John Chiarelli
Oral Qualifying Exam Syllabus

1. Combinatorics

- Enumeration: counting arguments, bijections, generating functions, binomial and multinomial coefficients, recurrence relations, inclusion-exclusion, Stirling's formula
- Set systems: Sperner's Theorem, LYM Inequality, Erdos-Ko-Rado, Dilworth's Theorem, statement of Kruskal-Katona
- Lattices: geometric and distributive lattices, chains in distributive lattices, linear extensions of posets, the Mobius inversion formula, Weisner's Theorem, Birkhoff representation theorem
- Correlation Inequalities: Harris-Kleitman, Fortuin-Kasteleyn-Ginibre (FKG), Ahlswede-Daykin
- Ramsey Theory: Ramsey's Theorem for graphs and hypergraphs, infinite Ramsey, Konig's Lemma, upper and lower bounds, probabilistic lower bounds, Hales-Jewett and van der Waerden
- Algebraic Methods: Combinatorial Nullstellensatz, Schwarz-Zippel lemma
- Basic Probabilistic Methods: Linearity of Expectation, Markov's Inequality, Chernoff bounds, Chebyshev Inequality, statement of Azuma's Inequality, binomial and Poisson distributions, applications to graph theory
- Method of Alterations: high girth and high chromatic number, R(k,k), independence number, lower bound on property B
- Lovasz Local Lemma: Symmetric and general versions, application to Ramsey lower bounds
- Entropy: Basic properties, Shearer's lemma

2. Graph Theorey

- Matching: Hall's Theorem, Konig's Theorem, augmenting paths, Tutte's theorem
- Connectivity and Spanning Trees: Menger's Theorem, Max Flow/Min Cut Theorem, Prim's Algorithm, Kruskal's Algorithm, Dijkstra's Algorithm, Matrix Tree Theorem, Cayley's Formula, Prufer Codes

- Property Thresholds: Properties of random graphs, applications of probabilistic methods to graph theory, Erdos-Renyi (threshold for connectivity), expected number of Hamiltonian paths in tournament, existence of core and applications
- Planarity: Euler's Theorem, proof that K5 and K3,3 are nonplanar, Kuratowski, Wagner's Theorem, Crossing number
- Coloring: Chromatic and Edge Chromatic Numbers, Brook's Theorem, Vizing's Theorem, 5-color Theorem, Galvin's proof of the Dinitz Conjecture
- Extremal Problems: Turan's Theorem, Statement of Regularity Lemma and its application to the Erdos-Stone Theorem, triangle removal lemma, Roth's theorem
- Path Construction: Dirac's Theorem, applications of longest path to graph properties

3. Computational Complexity

- The Computational Model: Turing machines, Church-Turing thesis, oblivious Turing machines, the universal Turing machine, the halting problem, Godel's Theorem, nondeterministic Turing machines
- Computational Complexity Classes: DTIME, P and NP, NP-completeness and reducibility, Cook-Levin Theorem, coNP, EXP, nEXP, Time Hierarchy Theorem, Ladner's Theorem, the polynomial hierarchy
- Diagonalization: oracle Turing machines, diagonalization techniques
- Space Complexity: SPACE and NSPACE, (N)PSPACE and (N)L, PSPACE and NL completeness, Savitch's Theorem, NL=coNL
- Randomized Computation: BPTIME and BPP, RP and coRP, ZPP, Sipser-Gacs
- Interactive Proofs: Interactive proof systems and examples (graph nonisomorphism), completeness and soundness, dIP and IP, Arthur-Merlin proofs, IP=PSPACE
- Quantum Computation: Quantum mechanical properties, BQP, quantum superposition and qubits, EPR paradox, quantum computation operations, quantum circuits and simulating classical computation, Grover's search algorithm, Simon's algorithm