Oral Examination Syllabus

Algebraic Topology

1. Fundamental Group
   elementary theory, Van Kampen theorem and applications, homotopy lifting properties, covering spaces and relation to π_1-sets

2. Homology
   simplicial/singular/cellular homology, long exact sequences, excision, Mayer-Vietoris sequences, axiomatic viewpoint

3. Cohomology
   simplicial/singular/cellular cohomology, long exact sequences, excision, Mayer-Vietoris sequences, axiomatic viewpoint, universal coefficients and relation to Ext/Tor, de Rham cohomology, cup/cap products, orientations of manifolds, Poincare duality and other dualities

4. Characteristic Classes
   vector bundles, axiomatic definitions of Chern / Stiefel-Whitney / Pontryagin / Euler classes, construction via classifying spaces, calculation of cohomology of infinite Grassmanian

Automorphic Forms

1. Classical Modular Forms
   definition of holomorphic and Maass forms, the q-expansion principle, calculations with elementary examples (Eisenstein series, theta functions, etc.)

2. The Spectral Problem
   the relationship between spectral theory for the Laplacian on quotients of the upper half plane, spectral theory for the Laplacian on $GL_2^+(\mathbb{R})$, the decomposition for the right regular representation of $GL_2^+(\mathbb{R})$, and classical holomorphic and Maass forms

3. Discreteness of the Spectrum for the Compact Case
   the proof that the Laplacian has a discrete spectrum on a compact quotient of the upper half plane

Representation Theory

1. Basic Representation theory for $GL_2^+(\mathbb{R})$
   Peter-Weyl theorem, Lie group vs Lie algebra representations, smooth and K-finite vectors, passage to $(\mathfrak{g}, K)$ modules, classification of irreducible $(\mathfrak{g}, K)$ modules

2. Unitary Representations for $GL_2^+(\mathbb{R})$
   unitaricity results, intertwining integrals for complimentary series

3. Whittaker Models

References

1. Algebraic Topology (Hatcher)

2. Characteristic Classes (Milnor and Stasheff)

3. Automorphic Forms and Representations (Bump)