

Syllabus for the Oral Qualifying Exam

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1 Set Theory

1.1 Basic Set Theory

- Suslin's Problem, Aronszajn Trees
- $\diamond \Rightarrow \exists$ Suslin tree; $\text{MA} + \neg\text{CH} \Rightarrow \nexists$ Suslin tree
- L , \diamond in L
- Shoenfield's Absoluteness Theorem

1.2 Forcing

- Forcing basics and forcing theorems
- CH , $\neg\text{CH}$, \diamond
- Product Forcing, Easton's Theorem
- Iterated Forcing, $\text{MA} + \neg\text{CH}$
- Consequences of MA , Almost disjoint forcing
- Formulation of Solovay's Theorem, Proof for PSP and DC

1.3 Inner Model Theory

- $L[U], L[U] \models \text{CH}, \Delta_3^1$ -wellorder of the reals
- Strong, superstrong, Woodin cardinals
- Extenders
- Iteration Trees
- Woodin's Genericity Iteration
- κ measurable $\Rightarrow \Pi_1^1$ -Determinacy

2 Model Theory

- Compactness, Löwenheim–Skolem Theorem, Tarski–Vaught Test
- Quantifier Elimination
- \aleph_0 -Categoricity, Oligomorphic automorphism group
- Fraïssé Theory
- Ultrapowers
- Atomic, prime, homogeneous, saturated models
- Omitting types
- Order indiscernibles: Existence, Stability \Rightarrow true indiscernibles
- Morley Rank, Morley Degree
- κ -categoricity $\Rightarrow \aleph_0$ -stability
- \aleph_0 -stability \Leftrightarrow totally transcendental