Oral Qualifying Exam Syllabus

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1. Modular Forms
   (a) The Modular Group
      i. $SL(2,\mathbb{Z})$ and Congruence Subgroups
      ii. Fundamental Domains for $SL(2,\mathbb{Z})$ and congruence subgroups
      iii. Cusps and Elliptic Points
      iv. The invariant measure of $\mathcal{H}$ under $SL(2,\mathbb{R})$.
   (b) Modular forms
      i. Modular and cusp forms
      ii. Fourier expansions
      iii. The dimensions of $M_k(\Gamma(1))$ and $S_k(\Gamma(1))$
      iv. Eisenstein Series, the Dedekind η function, Δ, and the Jacobi triple product formula
      v. The Petersson inner product on $S_k(\Gamma(1))$.
      vi. The L-functions for modular form and functional equations
   (c) Hecke Operators
      i. The slash and Hecke operators on holomorphic functions
      ii. Coset representatives for $SL(2,\mathbb{Z})\backslash M_n(\mathbb{Z})$
      iii. Commutativity and self-adjointness of the Hecke operators
      iv. Hecke eigenforms and Fourier coefficients
      v. Euler products for Hecke eigenforms
   (d) The Rankin-Selberg Method
      i. The nonholomorphic Eisenstein series
      ii. Analytic continuations and Euler products for the product of two modular forms
2. Ergodic Theory
   (a) Transformations on probability spaces
      i. Measure preserving transformations
      ii. Invertible Extensions
      iii. The unitary operator and its spectral properties
      iv. Poincaré Recurrence
      v. Strong mixing
      vi. Weak mixing and equivalent definitions
      vii. Ergodic transformations and equivalent definitions
      viii. Invariant measures for continuous maps
      ix. Unique Ergodicity
      x. Weyl's equidistribution criterion
      xi. An ergodic proof of Weyl's theorem on equidistribution of polynomial sequences
   (b) Ergodic Theorems
      i. Mean ergodic theorem

    ii. Maximal inequality and maximal ergodic theorem
    iii. Birkhoff's pointwise ergodic theorem
  (c) Continued Fractions
    i. The Gauss map and Gauss measure
    ii. Consequences of ergodicity of the Gauss map
    iii. Badly approximable numbers
  (d) Geodesic Flow
    i. Hyperbolic geometry
    ii. Geodesic and Horocycle Flow
    iii. Ergodicity of geodesic flow

3. Analytic Number Theory

  (a) Poisson summation
  (b) The Mellin transform and the Γ function
  (c) The Phragmen-Lindelof principle and convexity bounds
  (d) The Riemann ζ function and Dirichlet L-functions
    i. Euler products
    ii. θ functions
    iii. Analytic continuation and functional equations
  (e) Dirichlet's theorem on primes in arithmetic progressions.
  (f) The Prime Number Theorem.