

Oral Qualifying Exam Syllabus

Richard Voepel
Fall 2015

Oral Exam Committee : J. Beck, H. Iwaniec, A. Kontorovich, S. Miller (Chair)

Primary Topic : Diophantine Analysis and the Geometry of Numbers

- Pythagorean Triples
- Infinite Descent and Fermat's Last Theorem for $n = 4$
- Systems of Linear Diophantine Equations
- Dirichlet's Approximation Theorem
- Orders of Approximation and Khintchine's Theorem
- Kronecker's Approximation
- The Farey Sequence and Hurwitz's Theorem
- Continued Fractions and the Law of Best Approximations
- A Continued Fraction for e
- Irrationality of $\zeta(3)$ and an Irrationality Measure
- The Positive and Negative Pell Equations
- Siegel's Lemma and Roth's Theorem
- The Markoff Spectrum
- Markoff Numbers and Markoff Forms
- Hall's Ray and Gaps in the Spectrum
- Uniform Distribution Modulo 1 and Weyl's Criterion
- Uniform Distribution of Sequences of Polynomial Values
- Characterization of Normal Numbers
- Minkowski's First Theorem on Convex Bodies
- Sum of Squares
- The LLL Algorithm and the Shortest Vector Problem

Secondary Topic : Transcendental Number Theory

- Liouville's Theorem and Liouville's Constant
- Roth's Theorem and Champernowne's Constant
- The Lindemann-Weierstrass Theorem and the Transcendence of e and π
- The Gelfond-Schneider Theorem and the Transcendence of $\sqrt{2}^{\sqrt{2}}$ and e^π
- The Six Exponentials Theorem
- The Schneider-Lang Theorem
- Mahler's Classification and Koksma's Classification

References

- [1] A. Baker. *Transcendental Number Theory*. Cambridge University Press, New York, NY, 1990.
- [2] T. W. Cusick and M. E. Flahive. *The Markoff and Lagrange Spectra*. American Mathematical Society, Providence, RI, 1989.
- [3] L. Kuipers and H. Niederreiter. *Uniform Distribution of Sequences*. Dover Publications, Mineola, NY, 2006.
- [4] S. Lang. *Introduction to Transcendental Numbers*. Addison-Wesley, Reading, MA, 1966.
- [5] L.J. Mordell. *Diophantine Equations*. Academic Press Inc., New York, NY, 1969.
- [6] M. R. Murty and P. Rath. *Transcendental Numbers*. Springer, New York, NY, 2014.