Oral Qualifying Exam Syllabus

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April 2016

Analytic Number Theory

- $\zeta(s)$ and $L(s, \chi)$
    - Functional equation, approximate functional equation
    - Zero-free region, PNT
- Primes in arithmetic progressions
    - Siegel-Walfisz theorem
    - Bombieri-Vinogradov theorem
- Sieve methods
    - Λ^2 sieve, Brun-Titchmarsh estimate
- Bilinear Techniques
    - Statements of the additive, multiplicative large sieve inequalities
- Exponential sums
    - Weyl's method, subconvexity for $\zeta(s)$
- Dirichlet polynomials
    - Integral, discrete mean value estimates
    - Large values, $\zeta(s)$ zero-density estimates

Spectral Methods of Automorphic Forms

- Harmonic analysis on **H**
    - Classification of hyperbolic motions
    - Hyperbolic Laplacian, Whittaker function
- Fuchsian groups and automorphic forms
    - Fundamental domains
    - Double coset decomposition, Kloosterman sums
    - Eisenstein series, cusp forms
- Statement of the spectral theorem
    - Discrete part
    - Continuous part
        - Analytic continuation, functional equation of Eisenstein series
        - Residual spectrum, Eisenstein transform
- Statement of Selberg's trace formula, Weyl's law

Geometry of Numbers

- Minkowski theorems
- Reduction
    - Definite quadratic forms
    - Indefinite binary quadratic forms, Pell's equation

    - Binary cubic forms
- Applications
    - Representations of integers by quadratic forms
    - Dirichlet's theorem on diophantine approximation

References

1. JWS Cassels, *An Introduction to the Geometry of Numbers*, Springer-Verlag, 1959.
2. H Iwaniec, *Spectral Methods of Automorphic Forms*, 2nd edition, AMS and RMI, 2002.
3. H Iwaniec and E Kowalski, *Analytic Number Theory*, AMS, 2004.