

Oral qualifying exam syllabus

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1 Primary topic: experimental mathematics

1.1 Numerics

- Maple coding, recursion
- Newton's method
- Ansaetze
- Lagrange interpolation, least squares, cubic splines
- Numerical integration: Newton-Cotes quadrature, Gaussian quadrature
- Iterative maps: fixed points and stability, logistic map
- ODEs: phase plane analysis, fixed points and stability, Picard-Lindelöf theorem, Euler's method (implicit and explicit), Runge-Kutta methods
- PDEs: Heat equation, Poisson equation, Laplace's equation, finite difference methods

1.2 Probability

- Basic definitions, conditional probability, Bayes' theorem
- Random variables: moments, conditioning, asymptotic normality, central limit theorem
- Binomial, Poisson, and geometric distributions
- Probability generating functions
- Stochastic processes: Poisson process, Markov chains, random walks, Brownian motion
- Mathematical finance: option pricing, arbitrage, Black-Scholes formula
- Simulation: Monty Hall problem, Birthday problem, Gambler's ruin

2 Secondary topic: combinatorics and graph theory

2.1 Combinatorics

- Enumeration: bijections, multinomial theorem, multisets, inclusion-exclusion, generating functions, recurrence relations, Stirling's formula
- Linear programming: weak duality, strong duality, (fractional) covering and packing
- Posets and lattices: Hasse diagrams, chains and antichains, Dilworth's theorem, distributive lattices, Birkhoff representation theorem, Möbius inversion
- Probabilistic methods: Linearity of expectation, Markov's inequality, Chebyshev's inequality, Lovász local lemma, method of alterations
- Hypergraphs: Sperner's theorem, LYM inequality, Erdős-Ko-Rado theorem, statement of Kruskal-Katona

2.2 Graph theory

- Basics: trees, bipartite graphs, Hamiltonian cycles and paths
- Planar graphs: Euler's formula, five color theorem, proof that K_5 and $K_{3,3}$ are nonplanar
- Matchings: König's theorem, Hall's theorem, Tutte's theorem
- Extremal: Mantel's theorem, Turán's theorem, Erdős-Stone theorem
- Ramsey theory: Ramsey's theorem, upper and lower bounds