Syllabus for Oral Examination

Lun Zhang

1 Probability and Stochastic Calculus

Martingales, Stopping times and Filtrations

Stochastic process, indistinguishable, modification, same finite-dimensional distributions
Filtration, right(left)-continuity of filtration, adapted process
Measurability and progressively measurability of stochastic process
Stopping time, optional time
Martingale, submartingale, supermartingale, local martingale
Optional sampling
First and Second submartingale inequalities
Upcrossing inequality, submartingale converge theorem
Doob's maximal inequality
Doob-Meyer decomposition
Continuous square-integrable martingales, quadratic variation

Brownian Motion

Brownian motion, construction of Brownian motion, Markov property
Reflection principle, distribution of first passage time
Distribution of Brownian motion and it's running maximum
Strong law of large number for standard Brownian motion

Stochastic Integration

Simple process, construction of stochastic integral with respect to square integrable martingale
Continuous semimartingale, Ito's rule
Martingale characterization of Brownian motion
Girsanov theorem, Novikov condition

Stochastic differential equation

Strong solution, existence and strong uniqueness
Gronwall inequality

Weak solution
Pathwise uniqueness, uniqueness in the probability law sense

2 Mathematical Finance

Basic matrial

Change of measure, independence lemma
Risk-neutral measure, Market price of risk equations
First and second fundamental theorem
Feynman-Kac Theorem
Black-Scholes-Merton equation, put-call parity

Options

Knock-out barrier option (up and out call)
Lookback option
Asian option
American option, American pereptual put option, American call option

Change of Numeraire

Domestic risk neutral measure, foreign risk neutral measure
Pricing product quoted in foreign currency
Zero-coupon bonds, forward measures
T-forward prices, pricing call option under T-forward measure

3 Parabolic equation

Heat equation

Fundamental solution
Solution for homogeneous and nonhomogeneous heat equation
Mean-value formula
Strong maximum principle, uniqueness, smoothness
Local estimates of derivatives
Energy methods, uniqueness, backward uniqueness

Second-Order Parabolic Equations

Parabolic equation, weak solution
Existence of weak solution: Galerkin approximation

Existence and uniqueness
Regularity of weak solution
Weak maximum principles, Harnack inequality, strong maximum principle

References

[1] Chung, K.L. A Course in Probability Theory, Third Edition

[2] Evans, L.C. Partial differential equations. AMS Provindence, 2010.

[3] Karatzas,I. Shereve, E.S. Brownian Motion and Stochastic Calculus, second edition, Springer

[4] Shereve, E.S. Stochastic Calculus of Finace II, Springer