Oral Qualifying Exam Syllabus

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1. Combinatorics and Graph Theory

1.1 Combinatorics:

Counting and set theory: binomial coefficients, recurrence relations, generating functions, inclusion-exclusion, Stirling's formula, Stirling number. Erdös-Ko-Rado, Fisher's inequality, Ray-Chaudhuri-Wilson.

Lattice and poset: Distributive and geometry lattices, Birkhoff representation theorem, matriod, Mobius function, Weisner's theorem, Dilworth, Sperner, LYM inequality, linear extension of poset, dimension of poset.

Correlation inequality: Harris-Kleitman, Fortuin-Kastekeyn-Ginibre, Ahlswede-Daylein, Shepp XYZ.

Mathching theory: Hall's thm, König's thm, fractional cover and fractional matching, mathching polytope.

Ramsey Theory: Ramsey, infinite Ramsey, König's Lemma, Van de Waerden

1.2 Probabilistic Methods:

Basis: linearity of expectation, Markov's inequality, Chebyshev's inequality, Cheinoff bound, binomial and Poisson distribution.

Alternations: Ramsey, Independent number, graph with high girth and high chromatic number.

Second moment method: Threshold function, subgraph, clique number.

Lovasz local lemma: Symmetric and general versions, application to Ramsey.

Poisson Paradigm: Janson's Inequality and application on chromatic number of $G_{n,1/2}$. Brun's sieve and application on EPIT.

Martingales: Edge and vertex exposure, Azuma's inequality and application on chromatic number.

1.3 Graph Theory:

Matching: Tutte's thm, stable matching

Connectivity: Menger's Thm, Max Flow/Min Cut, structure of 2-connected graphs, minimal spanning tree, Kruskal's algorithm.

Extremal Problems: Turan's Theorem, Regularity lemma and its application on the Erdös-Stone Theorem, Chvatal-Rodl-Szemeredi-Trotter
Planarity: Euler's Formula, Kuratowski, Wagner

Coloring: Chromatic and Edge Chromatic Numbers, Brook's Theorem , Vizing's Theorem, Thomassen's Theorem, 5-color theorem, Galvin's Theorem, perfect graphs: definition and statements of theorems

2. Computational Complexity
P v. NP: Definitions, reducibility, the Cook-Levin Theorem, NP completeness of SAT, Independent set, 0/1 integer programming, coNP, what if P=NP

Diagonalization: Ladner's Theorem, Oracle Turing Machines, Baker-Gill-Solovay Theorem

Space-bounded complexity: definitions, configuration graph, PSPACE completeness of TQBF, NL completeness of PATH, Savitch's theorem , Immerman-Szelepcsényi Theorem

Polynomial hierarchy: Definitions of Σ_i, Π_i, complete problems, ATM, AP=PSPACE, Time/Space tradeoff for SAT

Circuits: P $\subset$ P/poly, CKT-SAT and alternate proof of Cook-Levin, Characterization of P/poly as TMs with advice, Karp-Lipton Theorem, Meyer's Theorem, existence of hard functions, Nonuniform Hierarchy Theorem, definitions of NC_i, AC_i

Randomization: Definitions of RP, BPP and ZPP, ZPP$\subset$RP$\cap$coRP, Error reduction, Sipser-Gacs Theorem, BPP $\subset$ P/poly, BPP $\subset$ $\sum_2^p \cup \prod_2^p$, randomized reductions and definition of BP • NP

Interactive Proofs: definitions, dIP=NP, GNI $\in$ AM, NP completeness of GI implies $\sum_2 = \prod_2$, IP=PSPACE

Decision Trees: Decision tree complexity, 0- and 1-certificates, certificate complexity, randomized decision tree complexity, sensitivity, block sensitivity, degree, relationships between s(f),bs(f),C(f),D(f),deg(f),R(f)

Communication Complexity: Fooling sets, tiling lower bound, rank lower bound, Discrepancy , ε(f)