

Oral Qualifying Exam Syllabus  
Charles Wolf

Proposed Committee: Professors Kopparty(chair), Kahn, Saks, Saraf

**1. Combinatorics**

- a. *Basic Enumeration*: Counting arguments, generating functions, binomial coefficients, recurrence relations, inclusion-exclusion, Stirling's formula
- b. *Set Systems*: Sperner's Theorem, LYM Inequality, Erdos-Ko-Rado, Kruskal Katona, Dilworth's Theorem, Fisher's Inequality, Raychaudhuri-Wilson, Frankl-Wilson
- c. *Lattices*: Wiesner's Theorem, Distributive and Geometric Lattices, 1/3-2/3 Conjecture, Mobius Inversion
- d. *Ramsey Theory*: Ramsey's Theorem, Probabilistic Lower Bounds, Schur's Theorem, van-der Waarden
- e. *Correlation Inequalities*: Ahlswede-Daykin, FKG Inequality, Harris
- f. *Discrepancy*: Beck-Fiala Theorem
- g. *Polynomial Methods*: Cauchy-Davenport, Combinatorial Nullstellansatz, Kemnitz Conjecture

**2. Graph Theory**

- a. *Matching*: Hall's Theorem, Konig's Theorem, stable matchings, Tutte's Theorem
- b. *Connectivity*: Menger's Theorem, Max Flow-Min Cut,
- c. *Extremal Problems*: Turan's Theorem, Erdos-Stone, Szemerédi's Regularity Lemma
- d. *Coloring*: Vizing's Theorem, Brook's Theorem, 5 Color Theorem
- e. *Planarity*: Kuratowski's Theorem, Crossing Number

**3. Probabilistic Methods**

- a. *Basics*: Markov, Chebyshev, Chernoff Bounds, Linearity of Expectation, alterations, Azuma's inequality
- b. *Second Moment Method*: application to threshold functions containing a fixed graph
- c. *Local Lemma*: Symmetric and General Versions, Application to Ramsey Lower Bounds
- d. *Poisson Paradigm*: Janson Inequalities, Brun's Sieve

**4. Computational Complexity**

- a. P v. NP: Definitions, reducibility, the Cook-Levin Theorem, NP completeness of SAT, independent set, 0/1 integer programming, and directed hamiltonian path, conditions that imply  $P \neq NP$
- b. Diagonalization: Ladner's Theorem, Oracle Turing Machines and the Baker-Gill-Solovay Theorem
- c. Space-bounded complexity: definitions, PSPACE completeness of TQBF, NL completeness of PATH, Savitch's theorem, the Immerman-Szelepcsényi Theorem
- d. Separation theorems: Time and Space Hierarchy Theorems (deterministic and nondeterministic versions)

- e. Polynomial hierarchy: Definitions of  $\Sigma_i$ ,  $\Pi_i$ , complete problems, conditions that lead to the collapse of PH.
- f. Circuits: P/poly, CKT-SAT and alternate proof of Cook-Levin, Characterization of  $\subseteq$
- g. P/poly as TMs with advice, Karp-Lipton Theorem, Meyer's Theorem, existence of hard functions, Nonuniform Hierarchy Theorem, definitions of  $NC_i$ ,  $AC_i$
- h. Randomization: Definitions of RP, BPP and ZPP, error reduction, Sipser-Gacs Theorem, BPP/poly, randomized reductions and definition of  $BPP \subseteq \bullet NP$
- i. Interactive Proofs: definitions,  $dIP=NP$ , GNI AM, NP completeness of GI implies  $\Sigma_2 = \Pi_2$ ,  $IP=PSPACE$
- j. PCP theorem: definitions, equivalence of the 3 versions, hardness of approximation for vertex cover and independent set.