Oral Examination Syllabus
Andrei Zherebtsov
Major Topic: Partial Differential Equations

1. Laplace Equation
   a) The fundamental solution
   b) $\Delta u = f$ in Ω, Ω bounded domain, f Hölder continuous
   c) Mean value formula and maximum principles
   d) Harnack's inequality
   e) Green's function for a ball and for half space
   f) Perron's method
2. Sobolev Spaces
   a) Density of smooth functions in Sobolev spaces
   b) Global extensions
   c) Trace theorem
   d) Sobolev inequalities (Gagliardo-Nirenberg-Sobolev, Morrey)
   e) Compact imbeddings
   f) Poincaré's inequality
3. Second Order Elliptic Equations
   a) Strong and weak maximum principles
   b) Uniqueness of the Dirichlet boundary value problems
   c) Definition of weak solutions
   d) Existence: Lax-Milgram, energy estimates, Fredholm alternative
   e) Interior and boundary regularity of solutions
4. Classical solutions. Schauder approach.

Minor Topic: Functional Analysis

1. Hilbert Spaces
   a) Riesz lemma
   b) Orthonormal bases
   c) Dual of a Hilbert space
   d) Lax-Milgram Theorem
2. Hahn-Banach theorems(analytic and geometric forms)
3. Baire category theorem,principle of uniform boundedness, open mapping theorem, closed graph theorem
4. Weak topologies, weak * topology, Banach-Alaoglu theorem
5. Compact operators. Riesz-Fredholm theory. Spectrum of a compact operator
6. Spectral decomposition of compact autoadjoint operators.