Oral Qualifying Exam Syllabus

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Major Topic: Lie Algebras

- Definitions, examples, representations, modules
- Solvable, nilpotent, simple, and semisimple Lie algebras, and the Killing form
- Theorems of Engel and Lie
- Cartan's criteria for semisimplicity and solvability
- Semisimple Lie algebras as direct products of simple Lie algebras
- Complete reducibility of modules for semisimple Lie algebras
- Representations of $\mathfrak{sl}(2, \mathbb{C})$
- Root systems and axiomatics
- Simple roots and the Weyl Group
- Construction of root systems and automorphisms
- Universal enveloping algebras
- PBW theorem
- Free Lie algebras
- Generators and relations, Serre's theorem
- Finite dimensional modules

Minor Topic: Representation Theory and Symmetric Functions

- Basic definitions, examples, representation theory of finite groups
- Characters, orthogonality relations, Schur's lemma, complete reducibility
- Schur-Weyl duality
- Irreducible complex representations of $\mathfrak{S}_n$
- Partitions and Young diagrams
- Ring of symmetric functions
- Schur functions
- Inner products and orthogonality
- Macdonald polynomials

References

1. Humphreys, J. E. *Introduction to Lie Algebras and Representation Theory.* Springer-Verlag, New York, 1972.

2. Goodman, R., Wallach, N. R. *Symmetry, Representations, and Invariants.* Springer, 2009.

3. Macdonald, I. G. *Symmetric Functions and Hall Polynomials.* Second edition. Oxford University Press, New York, 1998.