Oral Qualifying Exam Syllabus

Zahra Aminzare

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Committee

- Eduardo Sontag (Chair)
- Daniel Ocone
- Richard Falk
- Young-Ju Lee

Dynamical Models in Biology

Part 1. ODE models

- Exponential growth, logistic equation
- The chemostat, Michaelis-Menten kinetics, stability analysis, global behavior
- Epidemiology: SIRS model, analysis of equations, R_0, stability, effect of immunizations
- Chemical Kinetics: formalism, stoichiometry; enzymatic reactions
- Quasi-steady state approximations and singular perturbation analysis
- Multi-stability due to positive feedback; cell differentiation and bifurcations
- Periodic orbits and limit cycles, Poincaré-Bendixson Theorem, van der Pol oscillator, Bendixsons Criterion
- Neurons, action potential generation, Hodgkin-Huxley and FitzHugh-Nagumo approximations

Part 2. PDE Models

- Conservation or balance principle
- Transport equation

- Chemotaxis
- Diffusion: time of diffusion, separation of variables, boundary conditions
- Probabilistic interpretation
- Steady-state behavior of PDEs
- Steady states for a diffusion/chemotaxis model
- Examples: facilitated diffusion, density-dependent dispersal
- Traveling wave solutions of reaction-diffusion systems

Numerical Analysis

- **Polynomial Approximation**
  1. Lagrange interpolation
  2. Cubic Hermite interpolation
  3. Piecewise polynomial approximation
  4. Some error results
- **Numerical Quadrature**
  1. (composite) Trapezoidal, Simpson's and midpoint rules
  2. Derivation and error formulas
  3. Basic results of Gaussian quadrature formulas
- **Numerical methods for ordinary differential equations**
  1. Derivation and error estimates for one-step methods (e.g., Euler's method)
  2. Multistep methods (examples of explicit and implicit methods)
  3. Predictor-corrector methods
  4. Consistency, stability, and convergence of multistep methods
- **Finite Difference methods for partial differential equations**
  1. Laplace's equation: 5 point difference scheme, error estimates using discrete maximum principle
  2. Simple difference approximations of time dependent equations (transport, heat, and wave equations)
  3. Error analysis by the maximum principle
  4. Von Neumann stability condition
- **Finite Element methods for elliptic partial differential equations**

1. Standard variational formulation of boundary value problems with Dirichlet or Neumann boundary conditions
2. Energy norm error estimates
3. Solution of the resulting matrix equations (existence and uniqueness of solutions, iterative schemes)
4. Construction of finite element subspace: dimension of the spaces, basis functions, degrees of freedom, barycentric coordinates
5. Error estimates for piecewise linear interpolation