

# Oral Qualifying Exam Syllabus

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## I. Major Topic: Fractal Sets and Dimensions

### 1. Introduction

- Definition of Hausdorff measure and Hausdorff dimension
- Basic properties
- Vitali covering theorem
- Relationship between  $n$ -dimensional Hausdorff measure and Lebesgue measure
- Calculation of Hausdorff dimension and measure for Cantor set

### 2. Self-similar sets and Hutchinson's theorem

- Contractions and Similitudes
- Invariant sets and self-similar sets
- Open set condition
- The Hutchinson's theorem

### 3. Apollonian packings

- Lower and upper bounds for the Hausdorff dimension of the residual set for the Apollonian packing
- The disk-packing constant

### 4. Basic density properties

- Upper and lower densities
- Regular and irregular sets
- Elementary density bounds

## II. Minor Topic: Linear Operator Theory

### 1. Bounded Linear operators on Hilbert spaces

- Some examples of bounded linear operators with estimates of norms

- Bounded linear functionals and the Riesz representation theorem
- Self adjoint and normal operators
- Compact operators

## 2. Spectral Theory

- Spectrum and resolvent
- Spectral radius
- Functional Calculus
- Spectral theorem for compact self adjoint operators
- Spectral theorem for compact normal operators
- Spectral theorem for bounded self adjoint operators

## 3. Trace class and Hilbert-Schmidt operators

## 4. Trace and determinant

# References

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- [B] Boyd, D.W., *The Disk-Packing Constant*, *Aequationes Mathematicae* 7, 182-193, 1971.
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- [GG] Gohberg, I., Goldberg, S., *Basic Operator Theory*, Birkhäuser, 1981.
- [GGK] Gohberg, I., Goldberg, S., Kaashoek, M.A., *Classes of Linear Operators* Vol.I, Birkhäuser, 1990.