Syllabus for the oral exam of Pablo Angulo

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1 Elliptic linear PDE

1-Sobolev spaces

- Definition of Sobolev and Holder spaces.
- Basic properties, traces and extensions.
- Embeddings of Sobolev and Holder spaces.
- Compact embeddings.

2-Linear elliptic second order operators

- Existence: Lax Milgram and Fredholm theorems.
- Regularity of solutions.
- Maximum principles.
- Eigenvalues for elliptic operators.

3-Elliptic operators on manifolds

- Definition of differential operator. Symbol of an operator.
- Pseudodifferential operators.
- Existence of a pseudo-inverse for differential operators.
- Elliptic complexes. Hodge theory. Poincare duality.

2 Differential topology

1-Foundations

- Differentiable Manifolds.
- Transversality.

- Critical points of smooth maps. Sard's theorem. Morse functions.
- Orientations and Integration on Manifolds.

2-Brouwer degree and intersection numbers

- Degree of a continuous map.
- Applications of degree. Bouwer fixed point, invariance of domain.
- Intersection numbers. Euler number of a vector bundle.

3-De Rham cohomology

- Poincare lemma.
- Mayer-Vietoris sequence.
- Poincare dual of a closed oriented manifold.

4-Morse theory

- Morse lemma.
- The Morse inequalities.
- Morse Homology. Poincare duality.