Syllabus for Oral Examination

ShiTing Bao

Major topic: Partial Differential Equations

- **Laplacian Equation**

    (1) Fundamental solution
    (2) Mean-value formulas
    (3) Properties of harmonic functions:
        maximum principle, smoothness, local estimates, Liouville's theorem,
        analyticity, Harnack's inequality,
    (4) Green's function for a ball and for a half plane
    (5) Energy methods

- **Sobolev Spaces**

    (1) Definition of Sobolev spaces
    (2) Approximation by smooth functions
    (3) Extensions
    (4) Sobolev inequalities:
        Gagliardo-Nirenberg-Sobolev inequality, Morrey's inequality
    (5) Compact embedding theorem

- **Second Order Elliptic Equations**

    (1) Definition of weak solutions
    (2) Existence of weak solutions:
        Lax-Milgram theorem, energy estimate and Fredholm alternative
    (3) Regularity
    (4) Maximum principles

Minor topic: Functional Analysis

- **Banach Spaces**

  (1) Linear normed spaces
  (2) Barie category theorem
  (3) Dual and reflexive spaces
  (4) Alaoglu theorem

- **Hilbert Spaces**

  (1) Riezs Lemma
  (2) Orthonormal sets and Bases
  (3) Bessel's inequality

- **Linear Operators**

  (1) Uniform Bounded Principle
  (2) Open Mapping Theorem and Closed Graph Theorem
  (3) Fixed Points Theorem
  (4) Strong, Weak and Weak* Convergence