Oral Qualifying Exam Syllabus – Andrew Baxter
Committee: Doron Zeilberger (chair)
Steve Ferry
James Lepowsky
Michael Saks

MAJOR TOPIC: ENUMERATIVE COMBINATORICS

1. **Basic Enumeration:** counting arguments, generating functions, recurrence relations, inclusion-exclusion, pigeonhole principle, Stirling numbers, Bell numbers, Catalan numbers, Eulerian numbers, Fibonacci numbers

2. **Partition Theory:** Graphical representations, application of generating functions to partition theory, restricted partitions and permutations, application of Gaussian polynomials to partition theory, Identities of the Rogers-Ramanujan type. Standard Young Tableaux.

3. **Lattices and Posets:** Distributive Lattices and Geometric Lattices, Fundamental Theorem of Finite Distributive Lattices (Birkhoff Representation Theorem), Dilworth's Theorem, Möbius Inversion, Weisner's Theorem, Binomial Posets.

4. **Hypergeometric identities:** fundamental theorem; Fasenmyer's algorithm; Zeilberger's algorithm; Wilf-Zeilberger pairs; companion identity and dual identities

5. **Computer algebra:** Maple programming.

References:

1. Stanley, *Enumerative Combinatorics, Vol. 1.*

2. Andrews, *The Theory of Partitions.*

3. Petkovsek, Wilf, Zeilberger, *A=B.*

4. Zeilberger, *Enumerative and Algebraic Combinatorics.*

5. Zeilberger, *Three recitations on holonomic systems and hypergeometric series.*

MINOR TOPIC: ALGEBRAIC TOPOLOGY

1. **Fundamental Group** Definition of Homotopy, Fundamental Group. Van Kampen's Theorem. Cell Complexes. Definition of Covering Space, Universal Covering. Homotopy Lifting Property. Classification of Covering Spaces.

2. **Homology** Simplicial, Singular, and Cellular Homology. Relative Homology. Excision. Degree of a map $S^n \to S^n$. Euler Characteristic. Mayer-Vietoris Sequences. Borsuk-Ulam Theorem, Brouwer Fixed Point Theorem, Lefschetz Fixed Point Theorem

3. **Cohomology** Definition of Cohomology. Universal Coefficient Theorem for Cohomology. Definition of Cup Product, Cross Product, Cap Product. Cohomology Ring. A Künneth Formula. Orientation Classes of Manifolds. Poincare Duality.

Reference: Allen Hatcher, *Algebraic Topology*