Oral Qualifying Exam Syllabus

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1. Algebraic Number Theory
   (a) Number Fields: Integral Bases, Different, Discriminant
   (b) Ramification, Splitting of primes
   (c) Ideal Class Group, Minkowski's Bound, Finiteness of class number
   (d) Dirichlet's Unit Theorem
2. Elliptic Curves
   (a) The Group Law and Isogenies of elliptic curves
   (b) Elliptic Curves over finite fields, The Hasse Bound
   (c) Elliptic functions and elliptic curves over $\mathbb{C}$
   (d) Elliptic Curves over global fields, The Mordell-Weil Theorem
3. Analytic Number Theory
   (a) Analytic properties of Dirichlet L-functions and the Riemann zeta function
   (b) Dirichlet's Class Number Formula for quadratic fields and Dirichlet's Theorem on Arithmetic Progressions
   (c) The Prime Number Theorem
   (d) The Prime Number Theorem for Arithmetic Progressions and the exceptional zero
4. Modular Forms
   (a) Modular Forms for the full modular group and its congruence subgroups
   (b) Holomorphic Poincare and Eisenstein Series
   (c) Hecke Operators
   (d) Automorphic L-functions